CSCE 619-600
Networks and Distributed Processing
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Renewal Process Theory IV

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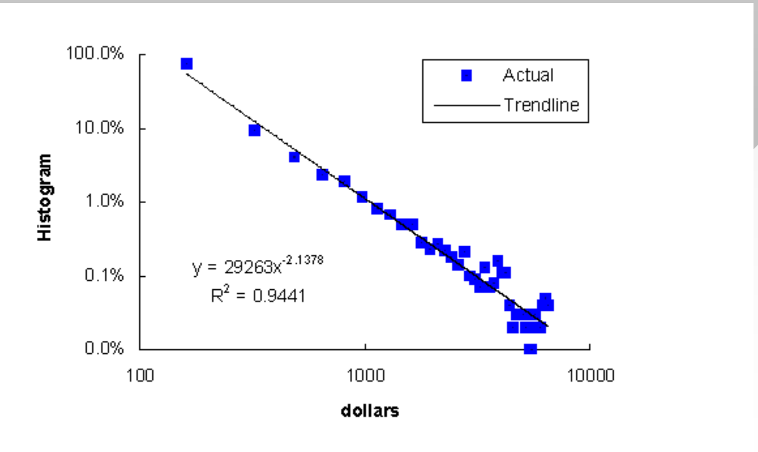
Agenda

- Homework #2 discussion
- Modeling examples
  - Isolation in P2P networks
  - Existing users in the system
  - Sampling age
  - Bank arrival models
  - Bank robber decisions
  - Sampling graphs
- Wrap-up

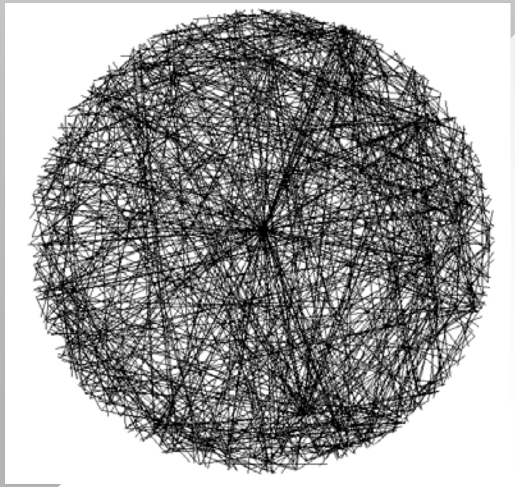
Homework #2

- Book problems
  - Let's solve them now
- Wealth problem: simple case
  - Lognormal distribution
  - Mean and variance of $\log(1+w)$ derived using the hints
- Wealth problem: complex case
  - You didn't have to get an exact Pareto distribution, but the tail of the PMF should have been approx Pareto



P2P Resilience

- Peer-to-peer networks have recently become an important Internet application
- After Napster's demise, P2P applications are either fully or at least partially distributed
  - P2P networks are dynamically formed at the application layer between the end users
  - Each user is called a “peer” and acts as a node in this virtual network
- To find desired files or content, search messages are sent over the graph of peers and are forwarded from one peer to the next



P2P Resilience 2

- There are many ways to route in P2P networks starting with random flooding as in Gnutella and finishing with complex algorithms of de Bruijn graphs
  - Our main concern here is not routing, but rather the resilience of these graphs to disconnection
- Each user acts as a router and must remain online for the network to function
  - Once a user departs, its place in the P2P graph must be either repaired or somehow replaced with another peer that can perform similar routing functions
- Denote by $L \sim F(x)$ the random lifetime of a user
- Initially a peer selects k uniformly random neighbors

P2P Resilience 3

- Isolation of v occurs at such random time T when its last neighbor leaves the graph
  - In such cases, v cannot find a replacement neighbor or repair its links
  - It thus must depart from the system
  - Clearly, this is undesirable
- Main metric of resilience is probability p that v can stay in the system for its entire lifetime L
  - Do not consider arrival of edges to v from other new nodes
- If neighbor replacement is allowed, this problem can be reduced to Markov chains (studied later)

P2P Resilience 4

- When replacement is not supported, the case is relatively simple and allows a closed-form solution
  - Straightforward application of renewal processes
- In hw #3, we obtain p and examine how it changes with k and parameters of the lifetime distribution
  - Does having Pareto $F(x)$ increase your resilience compared to exponential?
  - If you double the number of neighbors, how does p behave?
- Question: assume Y is the duration that a neighbor stays online after v has selected it
  - What's the distribution of Y?

Existing Users

- Suppose you observe a large system of users at some random point $t \gg 0$ when it has $N(t) \gg 0$ users
- What is the distribution of lifetime of live users?
  - In other words, let Z_j be the lifetime of the j-th live peer
- Define

$$G(x) = P(Z \leq x)$$

- How does $G(x)$ compare to $F(x)$?

Usage of Age

- Suppose each live user j can report its age $A_j(t)$ to any requesting peer
- Design an algorithm that can estimate the lifetime distribution $F(x)$ by querying live users
- Hints
  - Assume each user is a renewal process
  - System size is large (i.e., millions of peers)
  - Computation of numerical derivatives is available to you

Superposition

- Suppose n bank customers are driven by ON/OFF processes (ON = inside the bank, OFF = outside)
  - Let X_{ij} be the j-th inter-arrival delay of user i
  - Assume $E[X_{ij}] = C_i n$, where C_i is a random variable with some distribution $H(x)$
- Derive the distribution of delay between walk-ins at the bank assuming $n \to \infty$
  - What is the probability that more than 100 customers enter within a given hour?
- What is the probability that no customers arrive between $4{:}45$ and 5 pm?

Superposition 2

- Next, a bank robber needs Y seconds to clear out the safe entirely
  - Assume Y is exponential with rate μ
  - Also assume the robber grabs money at a constant rate s dollars per second (i.e., safe holds Ys dollars)
- However, if a new customer arrives while the robbery is in progress, the robber will stop and leave the bank with whatever money he has accumulated so far
- What is the expected pay off for the robber and what is the distribution of robbery duration T?

Superposition 3

- What is the probability that the bank loses less than $\$100$?
- Suppose the police response delay D is uniform in $[1,10]$ minutes from the time the robbery has started
  - What is the probability that they catch the robber?
- If the robber disables the alarm, but the police patrol the area using PASTA with mean delay 10 min
  - What is the probability they catch the robber?
  - Study two cases: 1) robber arrives just after the last police unit has departed; or 2) robber arrives randomly
- What is the best strategy for the robber when he can observe previous police cars?

Graph Example

- Suppose n users form a distributed graph
  - Degree X_j of user j is drawn from some distribution $F(x)$
- A new user v arrives into the system and performs a random walk over the existing graph
  - When the walk stops at node j, then j is marked as a potential neighbor of v
  - Assume the walk stops at j with probability proportional to its current degree:

$$P(j \text{ is marked}) = \frac{X_j}{\sum_{i=1}^{n} X_i}$$

- Suppose $Y_1, Y_2, \ldots$ are degrees of nodes marked by repeating this process
  - What is the distribution of Y_i? Compare $E[Y_i]$ with $E[X_j]$