CSCE 619-600
Networks and Distributed Processing
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Markov Chains

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Markov Chains

- We've already examined simple processes that exhibit the memoryless property
  - Exponential bus arrivals, or equivalently, Poisson processes are just one type of memoryless processes
- Definition: process $X(t)$ is called memoryless (Markov, or Markovian) if conditioning on $X(t_0)$ it follows that $X(t_0+s)$ for $s > 0$ does not depend on $X(t)$ at any $t < t_0$
- The future process $X(t_0+s)$ can only depend on the current state $X(t_0)$, but not the past

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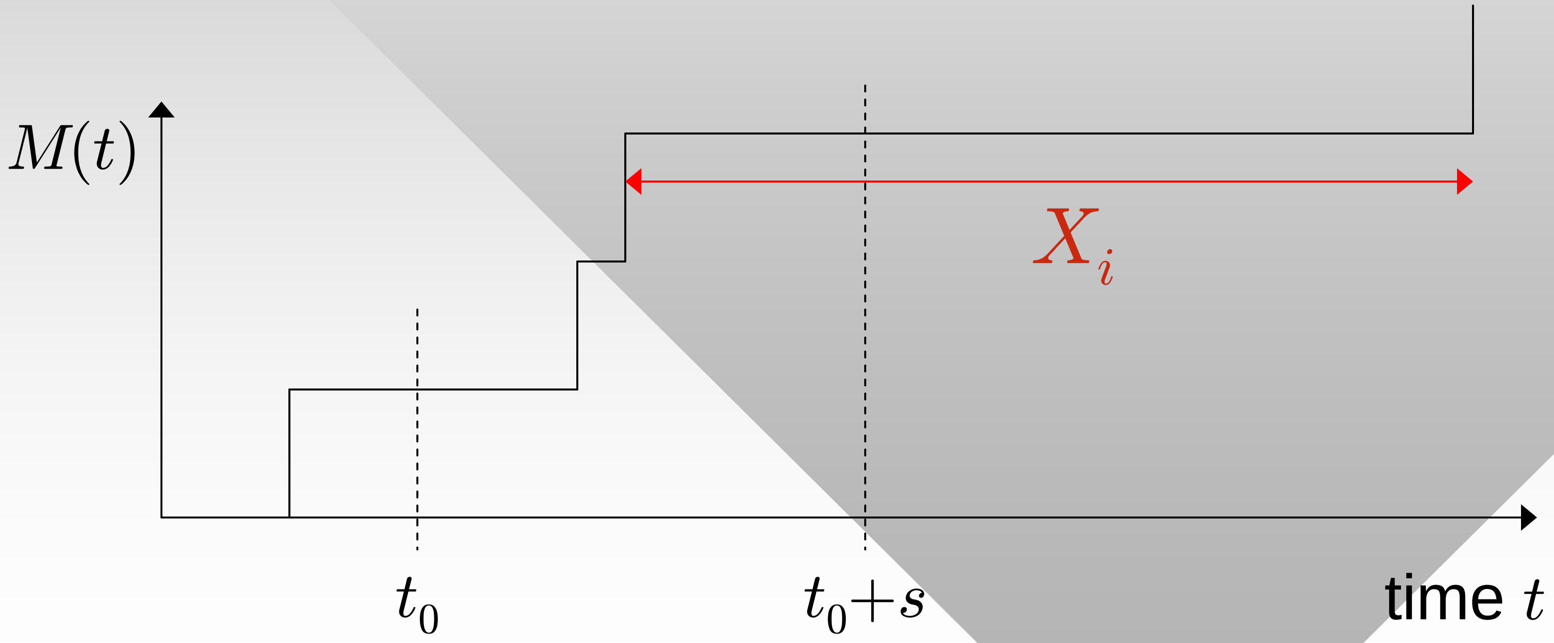
- Example: Poisson process $M(t)$
  - Why is it Markov?

$$M(t_0+s) = M(t_0)+Y(s), \text{ where } Y(s) \sim Poisson(\lambda s)$$



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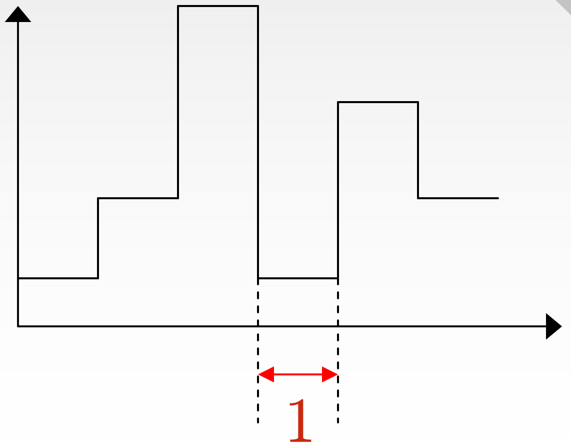
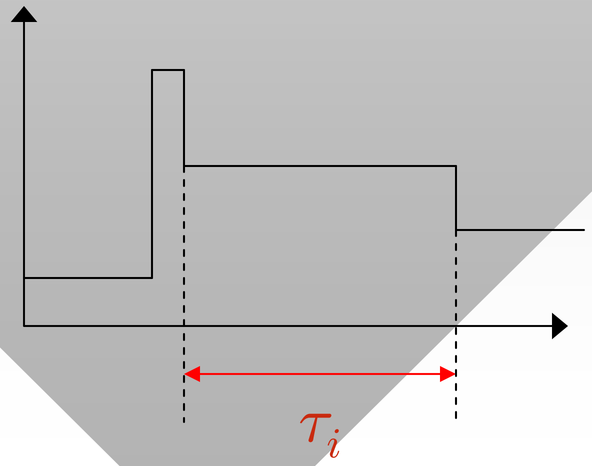
- In general, many phenomena exhibit oscillatory behavior and their models would benefit from random transitions that include both increase and decrease phases
- We will study two such processes
  - Discrete-time (left) and continuous-time (right)





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- Discrete Markov processes jump at every integer point $t = 0,\ 1,\ 2,\ \ldots$
  - Continuous-time chains spend a random amount of time τ_i in each state before making the i-th jump to another random state
- We first focus on discrete Markov chains
- Define X_n to be the integer state of chain at time n
  - Thus, X_n is a discrete-state process
  - Both finite and infinite number of states is acceptable
- Example: $X_n =$ the number of heads in n coin flips
  - How many states are there?

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- Definition: an infinite sequence of integral-valued random variables $\{X_n : n = 0, 1, \ldots\}$ is a *Markov chain* if

$$P(X_{n+1} = j | X_n, X_{n-1}, \ldots, X_0) = P(X_{n+1} = j | X_n)$$

  - Transition probability from state X_n to state X_{n+1} does not depend on where the chain was before X_n

- We assume that transition probabilities from every state i do not change over time
  - Then we can form a matrix of transition probabilities:

$$p_{ij} = P(X_{n+1} = j | X_n = i) = P(X_1 = j | X_0 = i)$$