CSCE 619-600
Networks and Distributed Processing
Spring 2017

Random Graphs II

Dmitri Loguinov
Texas A&M University

March 23, 2017

Agenda

- Midterm solutions
- Random-graph models
  - $G(n,p)$
  - $G(n,M)$

Random Graph Models 4

- Many of the early random-graph models stemmed from the work of Paul Erdos in the 1950s
  - One family of graphs is the famous $G(n,p)$ model
  - But we also study several other models with similar characteristics
- Definition: $G(n,p)$ is a random undirected graph with n nodes, where each edge (i, j) exists with an independent probability p
  - Goal of $G(n,p)$ is not necessarily to explain the Internet, but it serves as a good starting point
- Total edges in an undirected graph whose degree sequence is d_1, …, d_n?

$$\frac{1}{2}\sum_{i=1}^{n} d_i$$

Graphs 2

- Task: derive the distribution of degree d_i in $G(n,p)$
  - Assume that self-loops are not allowed, but this is not essential if the graph is large enough
- Possibly a simpler starting question would be
  - What is the average degree $E[d_i]$ in the graph?
- Assume that L_{ij} is an indicator variable that signals the existence of link (i, j)
  - Then:

$$L_{ij} = \begin{cases} 1 & (i,j) \in E \\ 0 & otherwise \end{cases}$$

Graphs 3

- Thus:

$$d_i = \sum_{j=1}^{n} L_{ij}$$

- We can further write that L_{ij} is a Bernoulli variable:

$$P(L_{ij} = 1) = \begin{cases} p & i \neq j \\ 0 & otherwise \end{cases}$$

- Thus, we have a binomial distribution for d_i:

$$d_i = B(n-1, p) \qquad E[d_i] = (n-1)p$$

Random Graph Models 5

- While the $G(n,p)$ model provides plenty of non-isomorphic graphs, there are certain families of graphs it cannot build
  - Example?
- Notice that every instance of $G(n,p)$ may have a different number of edges
  - To overcome this limitation, we have $G(n,M)$
- Definition: random graph $G(n,M)$ is a uniformly random instance among all undirected graphs with n nodes and exactly M edges

Random Graph Models 6

- How to construct a $G(n,M)$ graph?
- There is a total of $N = n(n-1)/2$ possible edges in an undirected graph
  - $G(n,M)$ uniformly selects M edges out of N
  - Each graph is built with an equal probability:

$$p_M = \frac{1}{\binom{N}{M}}$$

- Algorithm:
  - Create an array of N elements representing edges
  - Throw a uniform random point into this array, swap the selected edge with the last element, and shrink array by 1
  - Repeat the last step M times