Statistical Analysis and Distortion Modeling of MPEG-4 FGS

Min Dai
Electrical Engineering
Texas A&M University, TX 77843

Dmitri Loguinov
Computer Science
Texas A&M University, TX 77843

Hayder Radha
Electrical and Computer Engineering
Michigan State University, MI 48824
Background

• Rate-distortion (R-D) theory
  – The fundamental tradeoff in the design of any lossy compression system

A typical R-D curve

• R-D function:
  – A bound on achievable (or theoretically possible) distortion for a given coding rate
  – A powerful tool in Internet streaming
Background (cont.)

• Scalable coding is widely applied in Internet streaming
  – Provides the capability of recovering video information by partially decoding the compressed bitstream
  – Fine Granular Scalability (FGS) has been chosen in the MPEG-4 standard

• Fine granular scalability (FGS):
  – One low-bitrate base layer (BL) to provide a low, but guaranteed quality
  – One high-bitrate enhancement layer (EL) to provide fine quality improvement
  – EL can be truncated at any codeword
Motivation

• Current status:
  – No current closed-form R-D model can capture all the complexities of a real encoder
  – No specific work has been done on R-D modeling of scalable video coding for Internet streaming

• Goals in this paper:
  – Understand the statistical properties of FGS input and propose a more accurate statistical model for it
  – Study the bitplane coding process in FGS and derive a closed-form distortion model
Related work on Statistical Models

• Input to FGS EL:
  – DCT residue between the original image and the reconstructed image from BL

• The two most popular models of DCT residue:
  – Zero-mean Gaussian distribution:
    \[ f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \]
  – Laplacian distribution (double exponential):
    \[ f(x) = \frac{\lambda}{2} e^{-\lambda|x|} \]
Related work on Statistical Models

- The PMF of DCT residue with Gaussian and Laplacian estimations in frame 0 of the Foreman CIF sequence (left). Logarithmic scale of PMFs for the positive residue (right).
Proposed Statistical Model

- Mixture Laplacian model:

\[ f(x) = p \frac{\lambda_0}{2} e^{-\lambda_0|x|} + (1 - p) \frac{\lambda_1}{2} e^{-\lambda_1|x|} \]

where \( \lambda_0 \) denotes the small variance Laplacian distribution and \( \lambda_1 \) denotes the large variance Laplacian distribution.

- We use the Expectation-Maximization (EM) algorithm to obtain Maximum-likelihood (ML) estimation for parameters \( \{ p, \lambda_0, \lambda_1 \} \)
Results of Proposed Model

- The real PMF and mixture Laplacian (left) and the logarithmic scale of the positive part (right)
More Results

• The weighted absolute error of estimation in Foreman CIF (left) and Coastguard CIF (right)

All test sequences are coded at 10fps and 128 kb/s in the base layer
Current Distortion Models

• Classical model:

\[ D = \varepsilon^2 \sigma_X^2 2^{-2R} \]

where \( \varepsilon^2 \) is a signal-dependent constant, \( \sigma_X^2 \) denotes the signal variance and \( R \) is the bitrate

• A variation of the classical model (proposed by Chiang et al. in 1997):

\[ R = aD^{-1} + bD^{-2} \]

where parameters \( a, b \) are obtained empirically

• Distortion model for Uniform Quantizer (UQ):

\[ D(\Delta) = \Delta^2 / \beta \]

where \( \Delta \) is quantization step and \( \beta \) equals 12
Performance of Current Models

- Performances of current models in frame 0 (left) and frame 252 of Foreman CIF (right)
A more Accurate Distortion Model

• For each component in the mixture-Laplacian model, the distortion is:

\[ D_i(\Delta) = \frac{-1}{(1-e^{-\lambda_i\Delta})} \left\{ e^{-\lambda_i(\Delta-1)} \left[ (\Delta-1+\frac{1}{\lambda_i})^2 + \frac{1}{\lambda_i^2} \right] - \frac{2}{\lambda_i^2} \right\}, \; i = 0, 1 \]

• Final version:

\[ D(\Delta) = p \cdot D_0(\Delta) + (1-p) \cdot D_1(\Delta) \]

where \( \Delta \) is the quantization step of each bitplane in the FGS EL and \( p \) is the probability of Laplacian component 0.
Experimental Results

- The average absolute errors in Foreman CIF (left) and Coastguard CIF (right)
Conclusion

• This paper proposed an accurate statistical model for DCT residue

• Based on this statistical model, we derived a closed-form distortion function for FGS EL

• In summary, this paper provides a good starting point for further research on FGS streaming