Wealth-Based Evolution Model for the Internet AS-Level Topology

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Abstract— **In this paper, we seek to understand the intrinsic reasons for the well-known phenomenon of heavy-tailed degree in the Internet AS graph and argue that in contrast to traditional models based on preferential attachment and centralized optimization, the Pareto degree of the Internet can be explained by the evolution of wealth associated with each ISP. The proposed topology model utilizes a simple multiplicative stochastic process that determines each ISP's wealth at different points in time and several "maintenance" rules that keep the degree of each node proportional to its wealth. Actual link formation is determined in a decentralized fashion based on random walks, where each ISP individually decides when and how to increase its degree. Simulations show that the proposed model, which we call *Wealth-based Internet Topology* (WIT), produces scale-free random graphs with tunable exponent α and high clustering coefficients (between 0.35 and 0.5) that stay invariant as the size of the graph increases. This evolution closely mimics that of the Internet observed since 1997.**

I. Introduction

Recent studies show that real-life large-scale networks not only exhibit power-law degree distributions, but are highly clustered. Thus, a significant effort has recently focused on developing graph generators that are capable of constructing random networks with power-law degree distributions [1], [2], [4], [7], [11], [13], [21], [22], [26], [35], [36], [43], [48] and high clustering [7], [23]. Among the previous approaches, preferential attachment [4] and optimized-based construction [13] have become the two major paradigms for explaining the Internet topology. The former theory relies on the principle that each joining node attaches its links to existing nodes with a probability proportional to their current degrees. The main explanation behind this behavior is a premise that new users perceive large-degree nodes as being more "attractive" compared to low-degree nodes. The latter theory models node join as an optimization problem and argues that each joining ISP aims to solve a certain trade-off between the benefit of improved connectivity and the cost of adding new links.

As we discuss next, the existing evolution theories exhibit certain limitations in the context of the Internet AS-graph. While acceptable in certain cases (such as social networks), preferential attachment [4] is usually too restrictive to realistically model the Internet graph as it bases link formation *solely* on the degrees of existing nodes and places too much weight on ISP "popularity." From the practical perspective, it is clear that such complex factors as geographic location, technical feasibility, business strategy, and various economic considerations contribute to the evolution of each network rather than the attractiveness or size of other networks. Optimization-based topology models [13] are viable alternatives to preferential attachment that capture more diverse factors related to ISP peering; however, the lack of *mutuality* (i.e., a joining provider cannot attach to an ISP that does not wish to peer with it) and absence of *economic basis* for link formation (e.g., a joining network operator would not attach to an ISP close to bankruptcy, regardless of how well-connected the latter one is) make them potentially unrealistic as well.

In addition, both preferential attachment and optimization-based construction depend on the *global* knowledge of the system and always create random graphs using centralized information. While this is certainly not a problem during simulations (i.e., most generators are centralized), we argue that any theory that relies on global knowledge inherently fails to *explain* how the Internet could have reached its current stage given the fact that no single ISP has complete information about the AS graph. In preferential attachment, it is hard to conceive that new ISPs will test the probability p_i of connecting to each existing ISP i and then select the peering point exactly according to the ratio of degree d_i to the global sum $\sum_{k=1}^{n} d_k$, where n is the number of nodes in the system. As for the theory of optimization-based trade-off, the algorithm requires complete information about the *structure* of the graph (not just the degree of each ISP) and burdens each new node with an optimization process with complexity $\Theta(n^2)$, which is hardly possible in practice.

In this paper, we overcome the above limitations and complement the previous efforts by proposing a different theory for the structure of the Internet that relies on 1) principles of economic evolution that govern the degree of each ISP and 2) distributed random walks that determine the actual attachment decisions. While the main focus of this paper is to *understand* the evolution of the Internet, we also provide specific algorithms that can be used to create new graphs and test them against those observed in the Internet over the last decade.

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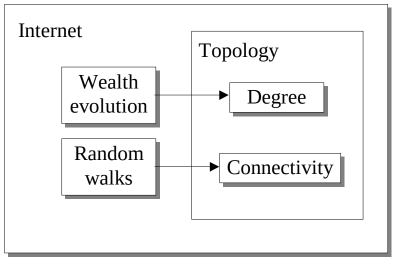


Fig. 1. Components of the wealth-based evolution model.

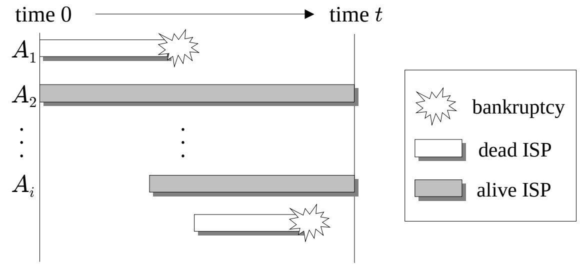


Fig. 2. Birth-death wealth evolution.

A. Degree

The structure of the proposed model is shown in Fig. 1, where the construction of the graph is driven by two paradigms – wealth evolution and random walks. As shown in the figure, the former is responsible for the degree distribution, while the latter for the formation of actual links. The main principle of the proposed model is that the *degree* of an ISP is a consequence of complex forces that can be macroscopically modeled by the wealth[1] of the ISP and not by the metrics found in the topology itself. This characteristic of the Internet makes it fundamentally different from other real-life graphs such as neural networks [4], [45], actor collaborations [4], scientific citations [34], [41], and numerous networks observed in physics [1], [9], [23], which also exhibit power-law degree distributions, but lack the financial orientation of the Internet.

Since individual and company wealth in many free-market societies is governed by Pareto distributions [32], we argue that the heavy-tailed degree of Internet ISPs is *a result of the particular structure of their wealth rather than anything else*. To understand this correlation, notice that it makes little sense to build topologies in which small local ISPs are modeled with extremely large degree, well-established backbone providers are assigned a handful of peering points, and the structure of individual companies evolves only based on the degree of other ISPs. Causality between company wealth and degree can be explained by many factors such as cost of link maintenance that makes higher degree more expensive, customer pressure that forces networks with many subscribers to be better connected, and the various QoS objectives that necessitate more peering points to deliver better service and extract more revenue from transit traffic; however, the exact specifics of this relationship are not essential and may be hidden under the umbrella of a simple economic model discussed below.

To capture the dynamics of open-market competition between the ISPs, our model assigns certain wealth $w_i(t)$ to each ISP i and acts on behalf of the ISP to keep its degree $d_i(t)$ proportional to its wealth. Individual ISP wealth $w_i(t)$ is governed by one of the simplest wealth evolution models that relies on random multiplicative increases/decreases in response to the stock market and various random economic decisions of the company. To account for bankruptcy that is prevalent among new startups, each ISP is removed from the system when its wealth drops below a certain threshold w_b needed to operate and provide service to its customers. This framework is illustrated in Fig. 2, where ISPs $A_1, A_2, \ldots$ sequentially join the system and compete in the Internet market using their individual assets $w_i(t)$ that include their initial funding, customer revenues, stock market gains, etc. As shown in the figure, at any time t, the system is composed of a random number of ISPs that are still alive (i.e., those that avoided bankruptcy), whose distribution of wealth determines the degree structure of the AS-level graph.

[1]Company wealth is an abstract concept that includes its revenue, customers, income, property and stock value, equipment, bandwidth, etc.

B. Links

For the construction of actual links, it is rather clear that the Internet evolves in a *distributed* fashion where the ISPs are not aware of any global characteristics of the network. To reflect the distributed nature of real attachment decisions, our model allows each ISP that plans to expand to perform random walks along the existing graph until it finds a neighbor that is willing to accept its peering request and satisfy its financial requirements (e.g., offer the right customer base, necessary economic model, and reasonable peering conditions). While the actual attachment decisions in real life are not haphazard, we argue that the event that a given ISP satisfies all of the above criteria for attachment may be modeled at some high level as *purely random*.

The final note is that our theory of random walks (as opposed to other means of finding neighbors) may be viewed as a by-product of the Internet market being a large social network, where many companies and individuals discover new acquaintances through existing links (i.e., business or personal relationships) rather than by randomly walking up to complete "strangers." This also allows our model to preserve locality where geographically close ISPs are more likely to peer.

We combine the above two methods (degree evolution and random walks) into a set of algorithms we call *Wealth-based Internet Topology* (WIT). Simulations show that WIT succeeds in producing power-law degree distributions with a flexible exponent α (including $\alpha = 1.2$ observed in the Internet) and is able to achieve levels of clustering close to those in the Internet (i.e., 0.45). More importantly, we find that the clustering coefficient of WIT matches that of the Internet during the *entire evolution* of the graph (i.e., as the size of the system increases) rather than for a single value of n as usually examined in prior work.

The remainder of the paper is organized as follows. We first review the background and related work in Section II. We then present our wealth evolution model in Section III and discuss the details of the topology construction algorithm in Section IV. Finally, we compare our model with existing methods in Section V and conclude the paper in Section VI.

II. Background and Related Work

In this section, we overview a small subset of related work and mention several well-known models that we study later in the paper.

A. Internet Topology and Power-law Degree Distribution

Faloutsos *et al*. [14] show that the Internet AS-level topology exhibits a power-law degree distribution, or the so-called "scale-free" phenomenon:

$$P(d_i > x) = (x/\beta)^{-\alpha}, \tag{1}$$

where d_i is the degree of node i, β is the scale parameter, and α is the shape parameter of the power-law distribution. Note that many similar observations [7], [10] are obtained from the archived snapshots of BGP (Border Gateway Protocol) routing tables collected by the Oregon Route View Server [31]. While sometimes it is argued that this data set does not reflect the whole view of the Internet, it has been reported in [10], [20] that graph properties such as the degree distribution are robust even with certain incompleteness in the data set of the Oregon Route View Server. In fact, it is shown in [10] that the number of links is the only difference between the graph inferred from the information collected by the Oregon Server and the one complemented with other sources (e.g., the Looking Glass tool and Internet Routing Registry database) and that the power-law degree distribution holds for both graphs. Note that similar observation can be found in [20] regarding the size of the largest connected component.

To model the scale-free property in the Internet, many efforts have been brought forward to design topology generators that produce power-law degrees. Some of them construct random graphs incrementally and others do not allow the growth of the network. We call the former algorithms *evolving* and the latter *non-evolving*. We next review two major classes of evolving models, i.e., preferential-attachment and optimization trade-offs, and follow it up with a discussion of non-evolving methods.

B. Preferential Attachment

The most common scale-free models used today are based on the theory of "preferential attachment" which is proposed by Barabási *et al*. [4] and implemented in their topology model *Barabási-Albert* (BA). At each discrete time step, BA adds a new node x to the graph, which is then randomly linked to $m \geq 1$ existing nodes using the preferential-attachment function:

$$p_i(t) = \frac{d_i(t)}{\sum_{k=1}^{n(t)} d_k(t)}, \tag{2}$$

where $p_i(t)$ is the probability that node i is selected for link formation at time t, $d_i(t)$ is its degree at time t, and $n(t)$ is the number of nodes in the graph at time t. This version of preferential attachment always produces graphs with shape parameter $\alpha \approx 2$. To relax the constraint on α, a method known as *Albert-Barabási* (AB) [1] adds the operations of link re-wiring and growth suspension (i.e., the graph evolves without adding new nodes).

Bu *et al*. [7] utilize shift-parameter $\lambda \in [-\infty, 1]$ in their model, which they call *Generalized Linear Preference* (GLP), and modify (2) to:

$$p_i(t) = \frac{d_i(t) - \lambda}{\sum_{k=1}^{n(t)} (d_k(t) - \lambda)}. \tag{3}$$

Through the use of (3), GLP achieves arbitrary values of $\alpha = 2 - \lambda \in [1, \infty)$ and high levels of clustering. Similar methods are proposed by Simon [6], [35], [36] and Krapivsky *et al*. [24], [25].

Other mechanisms in this category include *BRITE* [27] and *Inet* [21]; however, throughout this paper, we only study BA, AB, and GLP since their performance can be used to infer that of the other models.

C. Optimization-Based Models

Another major class in generating power-law degree distributions is first proposed by Carlson *et al*. [9] and later studied by Fabrikant *et al*. [13] in the context of the Internet. In their models called *Highly Optimized Tolerance* (HOT) each new node selects the attachment point based on the minimization of two objectives: the geographical length of the peering link and the average number of hops to other nodes in the graph. In particular, a new node i attaches to node k that minimizes the following:

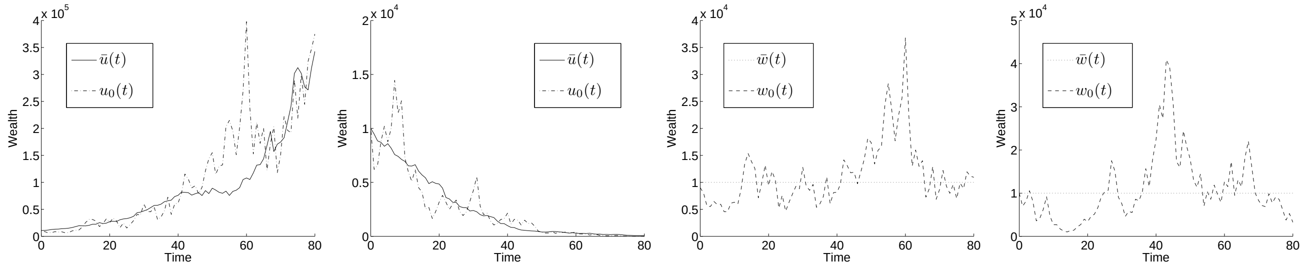
$$k = \arg\min_{j<i} \{\theta d_{ij} + h_j\}, \tag{4}$$

where d_{ij} is the Euclidean length of link (i, j), h_j is the average distance from j to other nodes in the graph, and θ is a parameter tuning the relative significance of the two objectives d_{ij} and h_j. Chang *et al*. [10] further explore optimization-based construction methods by allowing each AS to have multiple geographical locations, called *Points of Presences* (PoPs), where each new node i computes (4) by replacing d_{ij} with the minimum distance to all PoPs of node j.

D. Non-evolving Power-law Generators

In this category, we mention several generators that do not grow (evolve) the network over time. One of the simplest power-law graph construction models is called *Given Expected Degree* (GED) [11], [28], [29], [30]. GED is an extension of the classical Erdös-Rényi graph model $G(n, p)$ [12] in which edge-existence probability p is adjusted on a per-link basis to produce a heavy-tailed degree distribution. Specifically, a sequence of *weights* $\{w_i\}$ is first generated according to a Pareto distribution and then each edge (i, j) is created with independent probability:

$$p_{ij} = \min\left(\frac{w_i w_j}{D}, 1\right), \tag{5}$$



(a) unscaled wealth under explosion (b) unscaled wealth under collapse (c) scaled wealth under explosion (d) scaled wealth under collapse

Fig. 3. Illustration of wealth evolution ($w_0 = 10,000, n = 10,000$).

where $D = \sum_{k=1}^{n} w_k$. The min function is necessary since product $w_i w_j$ may exceed D, especially in sequences drawn from power-law distributions with shape parameter $\alpha < 2$.

A similar graph construction method called *Power-Law Random Graph* (PLRG) [2] replicates each node i exactly w_i times and then places random edges between the replicated nodes with equal probability. Thus, nodes with larger initial weight w_i receive proportionally more edges than nodes with smaller weight.

Additional non-evolving generators include random geometric graphs [22] and rewired small-world (Watts) networks that exhibit a heavy-tailed degree distribution [33], [45].

III. Wealth Model

We present our wealth evolution model in this section and proceed to show how it fits into our topology generator in the next section.

A. Wealth Evolution

According to the theory proposed in this paper, the Internet can be modeled as an economic entity, where ISPs dynamically join and leave the system based on random events. Denote by $w_i(t)$ the wealth of ISP i at time t. When a new ISP joins the system at time t_i, it comes with a certain amount of initial wealth w_0, which accounts for the startup capital obtained from venture capitalists.[2] During the lifetime of an ISP, it invests its wealth in business activities, retrieves financial return, and suffers losses, all of which allows its wealth $w_i(t)$ to randomly evolve over time.

Notice that the amount of investment return is usually proportional to the wealth of a company. Thus, we start with a basic *unscaled* model in which the individual investment-return cycle is a multiplicative stochastic process:

$$u_i(t) = \lambda_i(t) u_i(t-1), \text{ for } t > t_i, \tag{6}$$

where $u_i(t)$ is the unscaled wealth of user i at time t, $\lambda_i(t)$ is a random variable drawn from some distribution describing the randomness of the investment-return market cycle, and t_i is the join time of node i. We assume that $\lambda_i(t)$ is a stationary process that is independent among the ISPs.

[2]Our model uses fixed w_0; however, a simple extension to random startup funding is possible as well. Simulations show that such an extension produces almost identical results.

Note that in (6), we do not constrain the distribution of $\lambda_i(t)$ for generality of the model; however, such generality may result in a collapse or explosion of system wealth. Specifically, if $E[\lambda_i(t)] > 1$, the average wealth will grow to infinity as the system evolves. On the other hand, if $E[\lambda_i(t)] < 1$, the average wealth will diminish to zero. To keep system wealth at an equilibrium (see below for a discussion of the reasons for doing so), we counteract any possible inflation of wealth by scaling (6) and taking the result to be the *real* wealth of each ISP:

$$w_i(t) = \frac{u_i(t)}{\rho(t)}, \tag{7}$$

where $w_i(t)$ is the scaled wealth of user i at time t and $\rho(t)$ is a random process that we determine next.

As before, define $n(t)$ to be the number of ISPs in the system and $\bar{u}(t)$ to be the average unscaled system wealth at time t:

$$\bar{u}(t) = \frac{1}{n(t)} \sum_{i=1}^{n(t)} u_i(t).$$

Then, we have the following lemma.

Lemma 1: Defining $\rho(t)$ to be:

$$\rho(t) = \frac{\bar{u}(t)}{w_0}, \tag{8}$$

the average scaled wealth of the system

$$\bar{w}(t) = \frac{1}{n(t)} \sum_{i=1}^{n(t)} w_i(t). \tag{9}$$

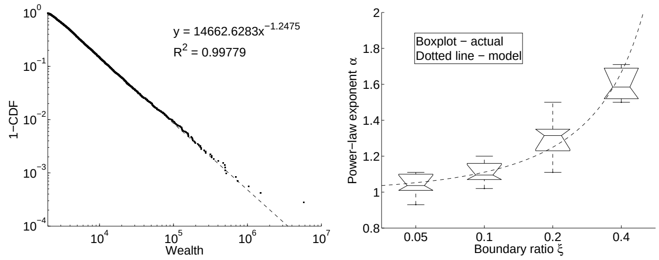
remains constant and equals w_0.

Proof: Substituting (7)-(8) into (9), we get:

$$\bar{w}(t) = \frac{\sum_{i=1}^{n(t)} u_i(t)}{\rho(t) n(t)} = \frac{\bar{u}(t)}{\rho(t)} = w_0, \tag{10}$$

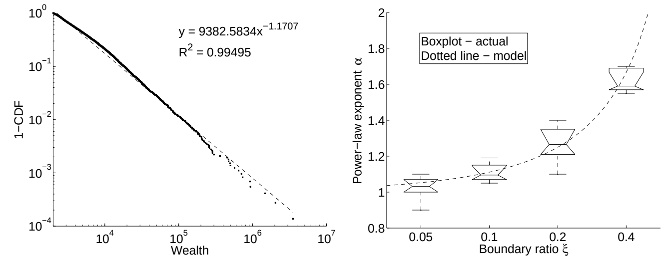
which produces the desired outcome. ■

Fig. 3 illustrates the effect of scaling in both explosion and collapse cases. In the first example shown in Fig. 3(a), we set $\lambda_i(t)$ to be uniformly random in $[0.55, 1.55]$ (i.e., $E[\lambda_i(t)] = 1.05$) and track the evolution of $u_0(t)$ together with that of the average unscaled wealth $\bar{u}(t)$ for $t = 80$ time units. The figure shows that the unscaled average wealth $\bar{u}(t)$ keeps increasing and becomes 35 times larger than w_0 at the end of the observation interval. If we apply $u_i(t)$ in



(a) $\xi = 0.2$ (b) effect of boundary ratio ξ

Fig. 4. Wealth distribution at time $t = 200$ under static join ($10,000$ joining ISPs).



(a) $\xi = 0.2$ (b) effect of boundary ratio ξ

Fig. 5. Wealth distribution at time $t = 200$ under dynamic join ($\mu = 50$).

a topology generator in which ISP degree is proportional to wealth, the resulting network will expand its combined wealth and connections unboundedly, which will eventually result in a complete graph. In the second case presented in Fig. 3(b), we conduct the same simulation except this time $\lambda_i(t)$ is uniformly random in $[0.45, 1.45]$ (i.e., $E[\lambda_i(t)] = 0.95$). As shown in the figure, $\bar{u}(t)$ drops toward zero steadily during the evolution process, which leads to the opposite effect and results in all ISPs completely losing their degree. To illustrate that this effect does not happen to $w_i(t)$, Fig. 3(c)-(d) plots scaled wealth $w_0(t)$ and the corresponding system-wide average $\bar{w}(t)$. The figure demonstrates that in both cases, $w_0(t)$ fluctuates around its fixed mean $\bar{w}(t)$ despite the explosion and collapse of the underlying unscaled wealth $u_0(t)$.

Given a model of ISP wealth represented by (7), we next discuss the conditions of bankruptcy and obtain the power-law exponent of $w_i(t)$ as a function of the bankruptcy boundary. As discussed in the introduction, we impose a lower boundary $w_b = \xi w_0$ on each ISP's wealth such that no one in the system is poorer than this baseline. For simplicity of discussion in the rest of the paper, we use metric $\xi \in (0, 1)$ as the ratio of the bankruptcy boundary w_b to the initial wealth w_0 of each joining ISP. Armed with Lemma 1 and bankruptcy definitions above, one can reduce (6)-(8) to static models of wealth evolution in economics [8], [16], [18], [19], [37], [40] and immediately obtain the following result.

Theorem 1: For sufficiently large t, the wealth evolution process $w_i(t)$ described by (6)-(8) achieves a power-law distribution with exponent:

$$\alpha \approx \frac{1}{1-\xi}, \tag{11}$$

where $0 < \xi < 1$ is the ratio of the lower boundary w_b of personal wealth to the initial wealth w_0.

B. Simulations

We confirm (11) under two different ISP join schemes. The first method, which we call *static*, forces all individual ISPs to start their wealth evolution processes at the same time. We plot in Fig. 4(a) the resulting distribution of $w_i(t)$ and observe that it is indeed power-law with an exceptionally good fit. We next vary ξ in simulations and generate $1,000$ instances of the evolution process to examine the correlation between ξ and α. Fig. 4(b) presents the resulting box-plot distribution of actual power-law exponents α and shows that their average value agrees with model (11) very well.

The second method, which we call *dynamic*, allows ISPs to join the system according to some arrival process of rate μ. We experimented with several join processes and observed no impact on the corresponding wealth distribution. Without loss of generality, the rest of the paper uses Poisson arrivals of rate μ ISPs per time unit and keeps the average number of ISPs that join the network by time t equal to μt. Simulations shown in Fig. 5 demonstrate that dynamic join also produces power-law wealth distributions and that the corresponding expected power-law exponent $E[\alpha]$ follows (11) very accurately.

With the result in (11), we are ready to present our topology model and show how the power-law wealth distribution affects topology generation in the next section.

IV. Topology Model

We start this section by introducing the role of wealth in our topology model, then present link-construction algorithms, and finally study the degree distribution and clustering properties of the resulting generator.

A. From Wealth to Topology

This subsection describes how the wealth model interacts with the topology generator and how it determines the degree evolution of the Internet.

In the hypothetical Internet market modeled in this work, the connectivity of an ISP to the rest of the network decides its usefulness to the customers. In order to maximize its revenue, an ISP tends to build as many links to other ISPs as possible. However, such link expansion is always limited by the wealth of the ISP since each link incurs a certain amount of expense, which represents the cost of purchasing routers and other equipment, leasing bandwidth, and maintenance. Therefore, an ISP builds additional links when it has spare wealth and similarly removes links when its wealth cannot sustain the expense of existing links. In this regard, we model the degree of an ISP as being proportionally dependent on its wealth and closely correlated with random fluctuations in $w_i(t)$.

Suppose the link between nodes i and j induces certain (potentially random) cost C_{ij}. Throughout the rest of the paper, we replace random variables C_{ij} with their expectation C and omit the discussion of random link cost as it produces very similar results. Denote by $z_i(t) = Cd_i(t)$ the expense induced by all links of node i. Then, whenever the wealth of an ISP i drops below its current link expense:

$$w_i(t) - z_i(t) < 0, \quad (12)$$

the ISP must remove some of the existing links to reduce expense $z_i(t)$ below its wealth $w_i(t)$. On the other hand, if the wealth of ISP i allows more links than it currently has:

$$w_i(t) - z_i(t) > C, \quad (13)$$

new connections are built until $z_i(t)$ reaches its wealth limit.

A direct result of the above mechanism for link adjustment is the linear mapping between ink expense and wealth:

$$z_i(t) \approx w_i(t), \quad (14)$$

which leads to the following result:

$$d_i(t) \propto w_i(t). \quad (15)$$

Recalling that the economic system presented in (6)-(8) produces a power-law wealth distribution, the next theorem follows immediately.

Theorem 2: For large enough t, the degree distribution of random graphs constructed under conditions (12)-(13) is power-law with exponent:

$$\alpha \approx \frac{1}{1-\xi}, \quad (16)$$

where constant ξ is the wealth boundary ratio.

Notice that conditions (12)-(13) may result in oscillatory link behavior, which is not the case in reality. To this end, we provide a dampening threshold T to relax conditions (12)-(13) to read:

$$w_i(t) - z_i(t) < -T, \quad (17)$$

and

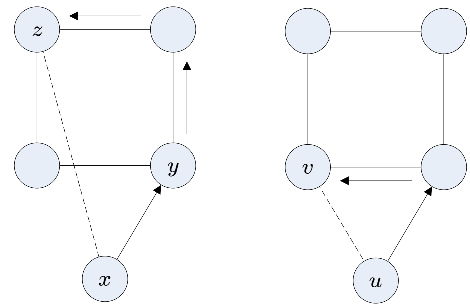
$$w_i(t) - z_i(t) > C + T. \quad (18)$$

By carefully choosing dampening threshold T, we are able to suppress the oscillations in link adjustment while allowing (16) to hold (see below for simulations).

We next focus on the link construction algorithm.

B. Topology Construction

As mentioned in the introduction, link addition in our topology generator depends on a simple set of rules based on random walks. When a new node x decides to enter an existing graph, it uniformly and randomly selects one existing node y in the network as the point of entry (while the assumption of uniformity may not be necessary, some degree of randomness is required). Once the node is introduced into the graph, it decides to "explore" its new location by performing a random walk of l steps. Once the initial walk stops at a node z, the new node x establishes a link to z, which is illustrated as the dotted line in Fig. 6(a). The above represents a selection process in which x searches for the first "acceptable" peering ISP. In the second phase of the join, node x starts from z and performs $m-1$ additional random walks to find the remaining



(a) initial attachment (b) link addition

Fig. 6. Random walks in topology generation.

$m-1$ neighbors, where m is determined by the initial wealth w_0 and the link price C. The decision to attach is determined solely by the final node where the walk stops and represents the random process where ISPs seek business partners with matching interests.

When an existing node u needs to build a new link, the procedure of finding a new neighbor is the same as in the initial attachment except that random walk start from u as shown in Fig. 6(b). On the other hand, when a node is forced to eliminate some of the existing links, it uniformly and randomly chooses a neighbor from its peering list and terminates the corresponding connection.

Finally, our topology model starts at time zero with a fully-connected core network of size m_0, which represents the initial stage of the Internet. Since simulations show that the actual setting of m_0 does not affect the properties of constructed graphs, we use the commonly suggested value $m_0 = 3$ in the rest of the paper.

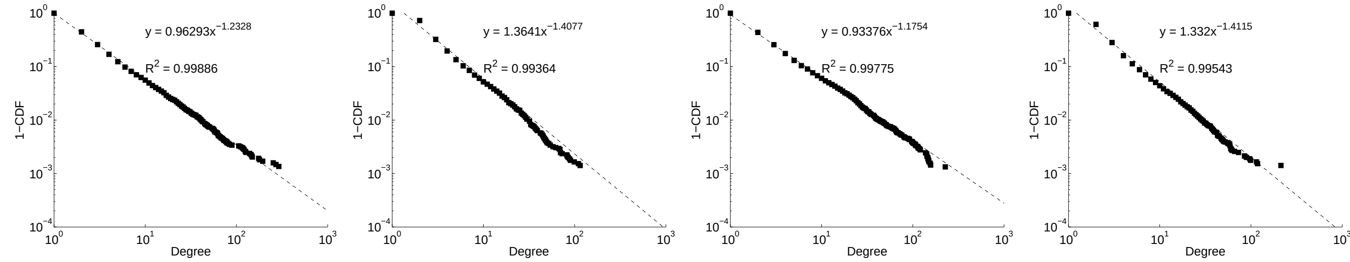
We refer to the set of algorithms described above as *Wealth-based Internet Topology* (WIT) and next study its degree distribution and clustering properties.

C. Degree Distribution

Our analysis of WIT in this subsection focuses on how degree exponent α is affected by four parameters: lower boundary w_b, link cost C, dampening threshold T, and walk length l. For convenience, we normalize the first three metrics by w_0 to obtain *lower-boundary ratio* $\xi = w_b/w_0$, *cost ratio* $c = C/w_0$, and *dampening ratio* $\tau = T/w_0$.

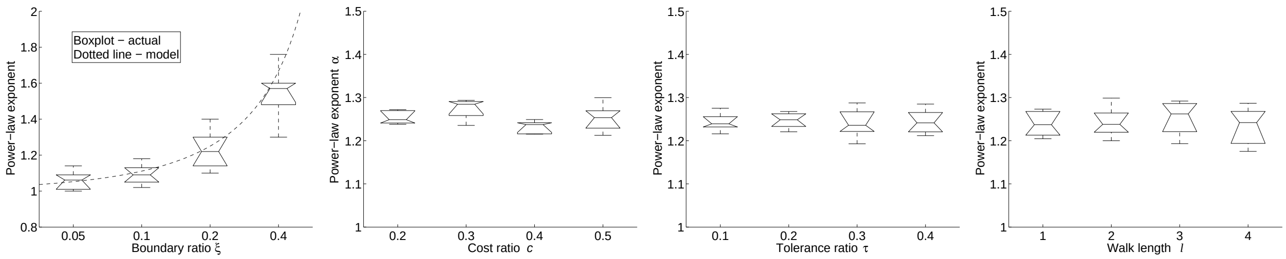
Before we begin, we first verify the quality of power-law distributions produced by WIT. Fig. 7 plots four examples of degree distributions obtained in simulations using several different values of l and ξ. This figure combined with additional results (not shown for brevity) indicates that WIT's degree exhibits very clear power-law tails that hold remarkably well for both short and long walks l.

Now, we are ready to examine the role of the four parameters introduced earlier in this subsection in the degree evolution process. To avoid confusion as to which parameter is responsible for which graph property, we study the effect of these factors separately by changing only one of them and keeping the other three fixed. We generate $1,000$ random



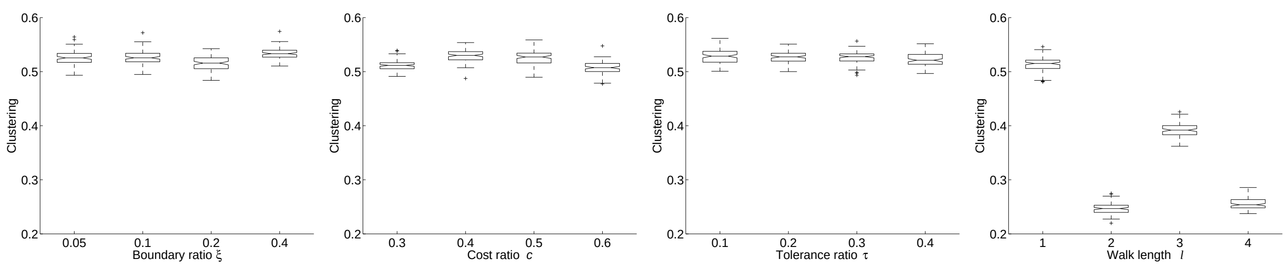
(a) $\xi = 0.2$ and $l = 3$ (b) $\xi = 0.4$ and $l = 7$ (c) $\xi = 0.2$ and $l = 15$ (d) $\xi = 0.4$ and $l = 100$

Fig. 7. Degree distributions of WIT graphs ($t = 200$, $\mu = 50$, $c = 0.4$, $\tau = 0.1$).



(a) effect of boundary ratio ξ (b) effect of cost ratio c (c) effect of dampening ratio τ (d) effect of walk length l

Fig. 8. Shape parameter α under $\xi = 0.25$, $c = 0.4$, $\tau = 0.1$, $l = 1$ ($t = 200$, $\mu = 50$).



(a) effect of boundary ratio ξ (b) effect of cost ratio c (c) effect of dampening ratio τ (d) effect of walk length l

Fig. 9. Clustering coefficient under $\xi = 0.2$, $c = 0.5$, $\tau = 0.1$, $l = 1$ ($t = 200$, $\mu = 50$).

graphs for each point in the figures and show the distribution of shape parameter α in the box-plots of Fig. 8. The results in Fig. 8(a) demonstrate that α increases as a function of ξ and follows model (16) very accurately. Additionally, Fig. 8(b)-(d) indicate that α is not sensitive to cost ratio c, dampening ratio τ, or walk length l, which also agrees with Theorem 2 very well.

Numerous additional simulations with different parameters show similar results (omitted for brevity) and conclusively establish that boundary ratio ξ is the *only* parameter that affects shape α of the degree distribution, which explains our global view of the model in Fig. 1. By tuning ξ, one can achieve arbitrary power-law exponents $\alpha \in [1, 2]$, and, as we show in the next subsection, tuning walk length l allows WIT to achieve flexible clustering $\gamma \in [0.008, 0.64]$.

D. Clustering

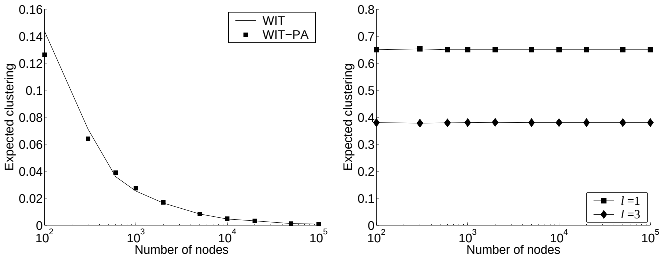
The clustering coefficient of a graph measures how frequently neighbors of a node connect to each other. Define T_i to be the number of triangles incident to node i. Then for a graph $G(V, E)$, its clustering coefficient is given by [44]:

$$\gamma(G) = \frac{1}{|V| - |V^{(1)}|} \sum_{i \in V - V^{(1)}} \frac{T_i}{d_i(d_i - 1)/2}, \qquad (19)$$

where $V^{(1)}$ is the set of degree-one nodes in G and d_i is the degree of node i.

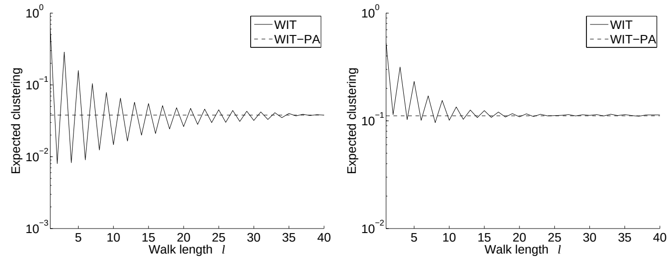
In what follows, we study clustering of WIT and show that it only depends on walk length l and not other parameters of the generator. Again, we conduct four sets of simulations as in the previous subsection and vary one of the four parameters in each set while keeping the others ones fixed. The box-plot of Fig. 9 shows clustering coefficients of WIT graphs generated under these conditions. In Fig. 9(a)-(c), the average clustering coefficient stays around 0.52 and does not exhibit much correlation with the change in the corresponding parameters. On the other hand, Fig. 9(d) shows that $\gamma(G)$ responds to walk length l, which we analyze in more detail next.

We start our discussion of how random walks determine local connectivity and clustering under the assumption that walk lengths are large, i.e., $l \gg 1$. Recall that random walks



(a) $E[\gamma(G)]$ for $l = 1,000$ (b) $E[\gamma(G)]$ for $l = 1$ and $l = 3$

Fig. 10. Clustering in WIT ($\xi = 0.2, c = 0.45, \tau = 0.3$).



(a) $E[\gamma(G)]$ for $n = 500$ (b) $E[\gamma(G)]$ for $n = 2,000$

Fig. 11. Effect of walk length on the clustering coefficient ($\xi = 0.2, c = 0.45, \tau = 0.3$).

on graphs represent the evolution of a stationary Markov chain. For a stationary chain the probability for a walk to terminate at node i is simply [47]:

$$p_i(t) = \frac{d_i(t)}{\sum_{k=1}^{n} d_k(t)}, \tag{20}$$

which is exactly the same as preferential probability (2). Therefore, we immediately obtain the following result.

Theorem 3: For walks longer than the mixing time of the corresponding Markov chain, WIT's clustering reduces to that of preferential attachment.

To validate Theorem 3, we implement a variant of our model that deploys preferential attachment instead of random walks in link construction. We refer to this variant as WIT-PA and compare it to pure WIT in simulations. By setting $l = 1,000$, we generate $1,000$ WIT graphs of different sizes n and plot their average clustering coefficients along with those of WIT-PA in Fig. 10(a). The figure shows that $E[\gamma(G)]$ of the two models is almost identical.

Analysis of short (i.e., significantly smaller than the mixing time of the chain) walks in WIT becomes non-trivial since the stationary distribution (20) does not hold. For $l = 1$, WIT always produces a lattice of triangles, where each node with degree $d_i \geq 2$ has $d_i - 1$ triangles. Therefore, without considering link deletion and rewiring, the clustering of node i is given by $2/d_i$, where d_i is the degree of node i. This immediately leads to:

$$E[\gamma(G)] = E[2/d_i] = \int_{\beta}^{\infty} \frac{2}{x} dF(x), \tag{21}$$

where $F(x) = 1-(\beta/x)^{\alpha}$ is the CDF of the power-law degree distribution. It follows from (21) that:

$$E[\gamma(G)] = \frac{2\alpha}{(1+\alpha)\beta},$$

which is a constant independent of the graph size n. Combining this with $\alpha = 1.2$ and $\beta = 1.45$ (obtained from simulations with $\xi = 0.2$, $c = 0.45$, $\tau = 0.3$, $l = 1$), we get $E[\gamma(G)] = 0.75$. The actual expected clustering of WIT for $l = 1$ is slightly lower and equals 0.64 as shown in Fig. 10(b), which can be explained by link deletion and rewiring not included in the above analysis.

For $l = 2$, new nodes produce mostly quadrangles instead of triangles and thus construct a poorly clustered graph, while for $l = 3$, WIT builds a mixture of triangles and pentagons, and exhibits lower clustering than with $l = 1$, but much higher than with $l = 2$. Fig. 10(b) plots WIT's average clustering coefficient for $l = 3$ and shows that it also stays constant as the graph evolves.

Alternating behavior in clustering between odd and even walk lengths is obvious for short walks and disappears when l becomes long enough. In Fig. 11(a), we show that WIT's clustering coefficient starts from 0.64 with $l = 1$, drops to 0.008 with $l = 2$, then oscillates with a decreasing amplitude, and finally converges to 0.038 as walk length l reaches 40. For larger n, Fig. 11(b) shows that when the system contains more "randomness" (i.e., $2,000$ nodes join the system), the clustering coefficient converges to its asymptotic value much quicker than in Fig. 11(a).

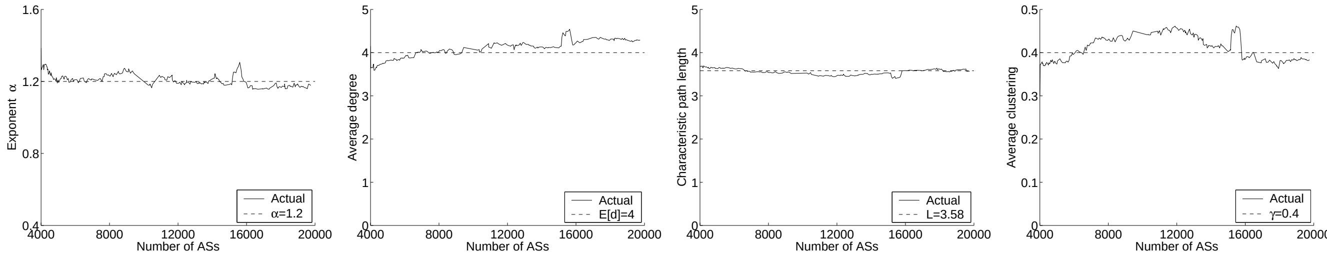
In reality, it is not hard to conceive that a new node in the Internet prefers short walks instead of long ones when deciding on link attachment. This in practice means that WIT equipped with short walks builds graphs with both constant and high clustering. Preferential attachment, on the other hand, only captures the behavior of long walks, produces small and decreasing clustering (see Fig. 10(a)), and thus cannot fully explain the structure of the Internet as we show next.

V. Evolutionary Comparison of Topology Models

With the topology model developed in the previous sections, we are now ready to answer the question of whether our model can track the evolution of the Internet. We start by understanding Internet's graph-theoretic properties as functions of time.

A. Analysis of the Internet Topology

In addition to the degree distribution and the clustering coefficient, the characteristic path length and average degree of a given graph are usually used in the evaluation of topology generators [4], [7]. A combination of these four metrics is usually sufficient to distinguish between most of the existing random graph models. Denote by $h(x, y)$ the hop distance between nodes x and y and by $h(x)$ the average distance from x to the rest of the graph. Recall that characteristic path length L is defined as the median of average distances over all nodes in the graph, i.e., $L = \text{median}_{x \in V}\{h(x)\}$ [44]. With the help of these metrics, we next explore how the Internet has evolved in the last 8 years.



(a) degree exponent α (b) average degree (c) characteristic path length L (d) clustering coefficient $\gamma(G)$

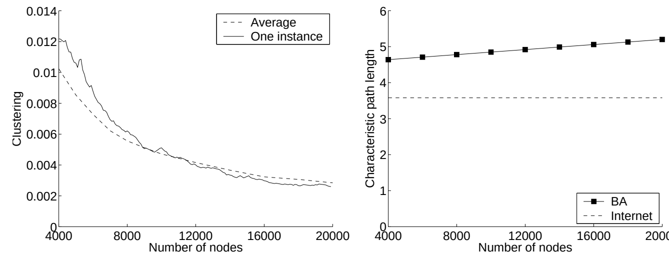
Fig. 12. Evolutionary view of the Internet topology observed between November 1997 and May 2005.

From the Oregon Route View Server [31], we collected 333 AS snapshots between November 1997 and May 2005 (one snapshot every ten days), extracted the corresponding AS topologies, and computed for each graph its average degree, power-law exponent α, clustering coefficient γ, and characteristic path length L. To examine the dynamic behavior of the Internet, we plot in Fig. 12 these graph properties against the size of the Internet. Fig. 12(a) shows that the degree distribution exhibits a constant power-law exponent, which is rather stable in $[1.15, 1.23]$. Besides the scale-free degree distribution, the Internet is almost invariant in its average degree and characteristic path length, which stay around 4 and 3.7 in Fig. 12(b)-(c), respectively. More interestingly, Fig. 12(d) indicates that the clustering coefficient of the Internet is not only high as reported in [7], [23], but also fairly constant between 0.35 and 0.47.

B. Comparison of Topology Models

It is possible for a topology generator to construct graphs that match the structural metrics of the Internet at a given time point (i.e., size n). However, as the Internet evolves and its size increases, graph properties of the generator may deviate from those in the Internet. Therefore, it is important to compare existing topology models from an *evolutionary* point of view, which tracks the corresponding graph metrics over the entire construction process. Fig. 12 shows that even though the size of the Internet keeps increasing over time, the four graph-theoretic properties remain invariant to the growth. The main question we aim to address in this section is *whether this invariance is captured by the existing generators?* We answer this question by examining how several existing models behave during the graph construction process and its evolution. Note that our work complements the previous efforts since it performs comparison analysis from a completely different perspective.

In particular, we compare WIT to several classical topology generators. For preferential attachment, we use BA, AB, and GLP. For optimization-based algorithms, we study HOT and allow each new node to link to $m \geq 2$ peering points. For non-evolving models, we modify GED to support incremental construction, where each new node joins the system and builds links to existing nodes with the probability described by (5). We refer to our version of GED as *Evolving GED* (EGED).



(a) clustering $\gamma(G)$ (b) characteristic path length L

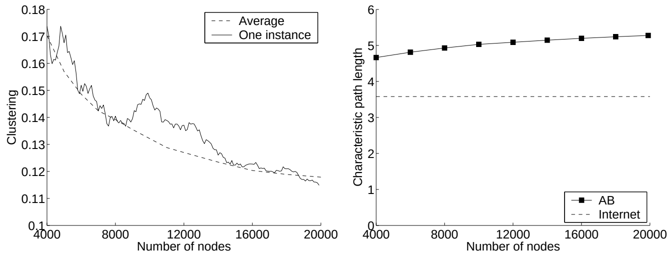
Fig. 13. Evolution of BA.

In our simulations of these generators, we attempt to ensure that the average degree and exponent α are the same as in the Internet (i.e., 4 and 1.2, respectively). Considering that BA and HOT always produce fixed $\alpha \approx 2$, we allow this rule to be violated in BA and HOT, but guarantee full conformance of the two metrics in the remaining models examined in this paper. Further note that AB and EGED usually produce disconnected graphs, in which case, we examine graph properties of the largest connected component of the corresponding graph.

In our first simulation, we let each generator build a random graph with $20,000$ nodes. During graph construction, we record snapshots of the partial graphs at different time epochs and compute their clustering coefficients and characteristic path lengths. We omit the snapshots of small graphs to be consistent with the size of the Internet whose structure before 1997 is not currently available.

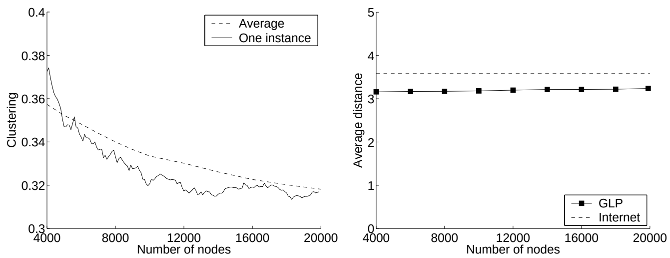
As we show below, oscillation in the clustering coefficient exists in all studied generators. To augment the information provided by a single instance of each stochastic process, we also show the expected clustering coefficients in all studied methods. For each generator, we create $1,000$ random graph evolutions and average the clustering coefficient at each time t. All figures below plot instant clustering as solid lines and their expected values as dotted curves.

Fig. 13(a) shows that BA exhibits very small clustering coefficients, which decay towards zero as the graph grows in size. This is clearly not representative of the situation in the Internet in Fig. 12(d). The characteristic path length of BA grows from 4.6 to 5.1 as shown in Fig. 13(b). Compared to the Internet where L is around 3.7, BA tends to push new nodes away from the center of the graph and thus produces



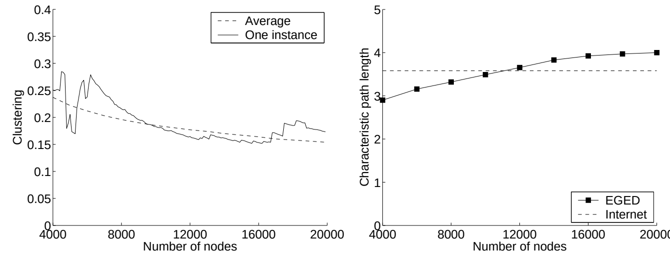
(a) clustering $\gamma(G)$ (b) characteristic path length L

Fig. 14. Evolution of AB.



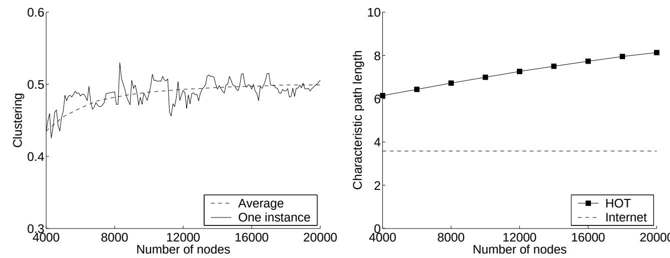
(a) clustering $\gamma(G)$ (b) characteristic path length L

Fig. 15. Evolution of GLP.



(a) clustering $\gamma(G)$ (b) characteristic path length L

Fig. 16. Evolution of EGED.



(a) clustering $\gamma(G)$ (b) characteristic path length L

Fig. 17. Evolution of HOT.

substantially larger characteristic path lengths than those in the Internet. AB improves over BA in terms of clustering as shown in Fig. 14(a); however, its $\gamma(G)$ also decreases as the graph size increases. The characteristic path length in AB is similar to that of BA as shown in Fig. 14(b).

Among the three preferential attachment methods, GLP shows the best clustering in Fig. 15(a) reaching as high as 0.37 for $n = 4,000$. However, as n increases to $20,000$, $\gamma(G)$ drops to 0.32, which is a common drawback of all examined preferential attachment methods, i.e., the clustering coefficient decreases as $n \to \infty$. In Fig. 15(b), the characteristic path length of GLP stays constant at 3.2, which is slightly smaller than that in the Internet.

In the category of non-evolving methods, EGED also demonstrates decaying clustering in Fig. 16(a) and keeps its $\gamma(G)$ significantly smaller than that of the Internet. Its characteristic path length starts from a small value of 2.9, but then becomes larger than that of the Internet at the end of graph evolution as shown in Fig. 16(b).

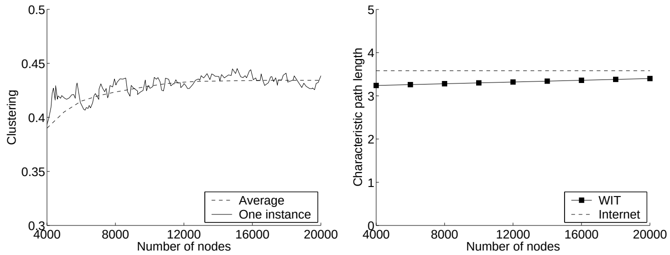
Interestingly, HOT exhibits in Fig. 17(a) very high clustering, which oscillates around 0.49. However, its characteristic path length is much higher than in the Internet and increases from 6.1 to 8.1 almost as a linear function of n as shown in Fig. 17(b). The preference of HOT for geographically short links leads to a graph that spreads out over the entire coordinate plane and thus results in a significantly larger characteristic path length than needed to model the Internet.

Similar to HOT, WIT displays high clustering during the entire graph evolution as shown in Fig. 18(a). The average clustering starts from 0.39 for $n = 4,000$ and converges to 0.43 for $n = 20,000$. Instant clustering oscillates around the dotted line in Fig. 18(a) and at certain points reaches as high as 0.45, which closely mimics the random fluctuation of γ in the Internet. In addition to producing flexible α, WIT is different from HOT in terms of characteristic path length. WIT's metric L is initially small, but eventually converges from below to 3.7 observed in the Internet as shown in Fig. 18(b).

Based on a combination of clustering coefficients and characteristic path lengths, one must conclude that graphs constructed by WIT are the closest to the Internet's evolutionary structure among the compared models. We also believe that WIT is a more realistic framework than some of the existing methods as it relies on distributed construction rules and allows each ISP to independently select its peering points based on internal factors such as its wealth, customer base, and QoS requirements that do not depend on the parameters or decisions of other ISPs.

VI. Conclusion

In this paper, we presented an alternative theory of the Internet evolution and developed a new topology generator based on wealth evolution and random walks. We showed that the generated graphs exhibited power-law degree distributions with flexible α and high, non-decreasing clustering coefficients. The characteristic path length of the proposed model was also close to that of the Internet and demonstrated invariance with respect to n. This combination of WIT's properties indicates that the proposed topology algorithm is viable in explaining the structural evolution of the Internet, at least to the extent possible in a very simple model. Future work includes extension of WIT with randomized parameters, derivation of theoretical expressions for WIT's clustering coefficients, and analysis of its spectral properties.



(a) clustering $\gamma(G)$ (b) characteristic path length L

Fig. 18. Evolution of WIT.

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