

# Modeling Residual-Geometric Flow Sampling

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# Agenda

- Introduction
- Underlying model of residual sampling
- Analysis of existing estimators
- Proposal of new estimators
- Performance evaluation
- Conclusion

# Introduction

- Traffic monitoring is an important topic for today's Internet
  - Security, accounting, traffic engineering
- It has become challenging as Internet grew in scale and complexity
- In this talk, we focus on two problems in the general area of **measuring flow sizes**
  - Determining the number of packets of elephant flows
  - Recovering the distribution of flow sizes

## Related Work

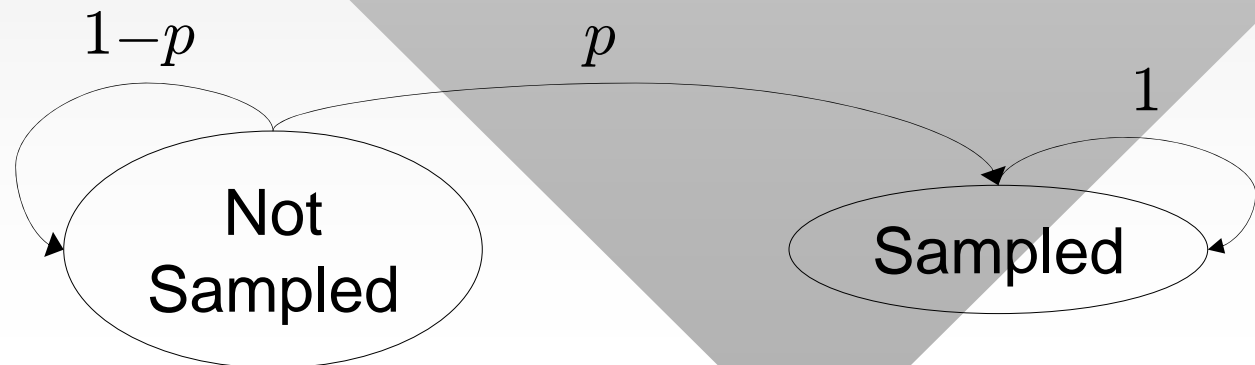
- Packet sampling
  - Sampled NetFlow (Cisco)
  - Adaptive NetFlow (Estan, SIGCOMM'04)
  - Sketch-guided sampling (Kumar, INFOCOM'06)
  - Adaptive non-linear sampling (Hu, INFOCOM'08)
- Flow sampling
  - Sample-and-hold (Estan, SIGCOMM'02)
  - Flow thinning (Hohn, IMC'03)
  - Smart sampling (Duffield, IMC'03/SIGMETRICS'03)
  - Flow slicing (Kompella, IMC'05)

# Analysis of Underlying Model

- Our talk is based on the sampling method proposed by sample-and-hold (Estan, SIGCOMM'02)
- We call this method by **Residual-Geometric Sampling (RGS)** due to two reasons:
  - This belongs to the class of residual-sampling techniques (Wang, INFOCOM'07/P2P'09)
  - It can be modeled by a geometric process
- Our analysis of RGS covers two goals:
  - Providing a unifying analytical model
  - Understanding the properties of samples it collects

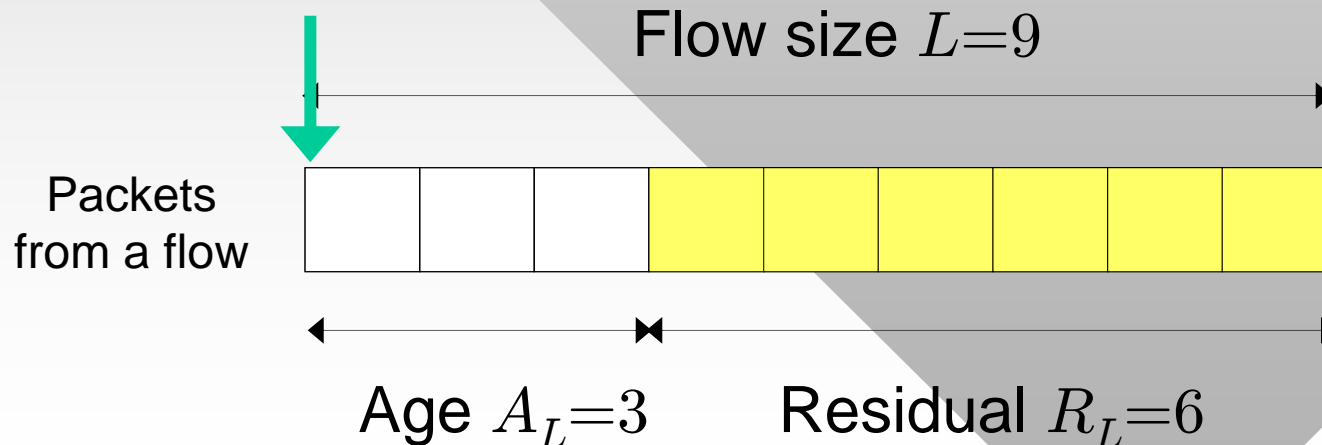
## Analysis of Underlying Model 2

- How does RGS work?
  - For a sequence of packets traversing a router, it checks each packet's flow id  $x$  in some RAM table
  - If  $x$  is found, its counter is incremented by 1
  - Otherwise, an entry is created for  $x$  with probability  $p$  and this packet is discarded with probability  $1 - p$
- The state of a flow can be modeled by a simple geometric process



# Analysis of Underlying Model 3

- We need several definitions:
  - Assume that flow sizes are i.i.d
  - Given a random flow with size  $L$ , define geometric age  $A_L$  the number of packets discarded from the front
  - Define geometric residual  $R_L$  the final counter value
- A flow of size 9 is not sampled until the 4<sup>th</sup> packet



# Analysis of Underlying Model 4

- Assume flow size  $L$  has a PMF  $f_i$ :

$$f_i = P(L = i)$$

- *Lemma 1*: Probability  $p_s$  of a flow being selected by RGS is:

$$p_s = 1 - \sum_{i=1}^{\infty} f_i (1 - p)^i$$

- *Lemma 2*: PMF  $h_i$  of geometric residual  $R_L$  can be expressed as:

$$h_i = \frac{p \sum_{j=i}^{\infty} f_j (1-p)^{j-i}}{p_s}$$



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## Previous Method – Single-Flow Usage

- Prior work on RGS (Estan, SIGCOMM'02) suggested following estimator of single-flow size:

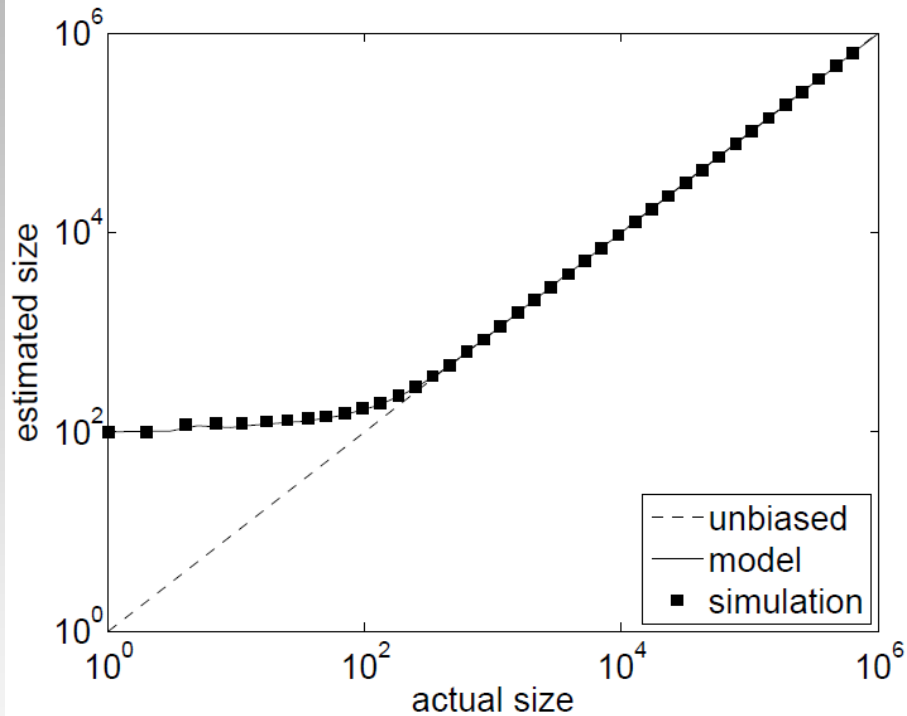
$$e(R_l) = R_l - 1 + 1/p$$

- *Theorem 1*: For given size  $l$ , the expected value of estimator  $e(R_l)$  is:

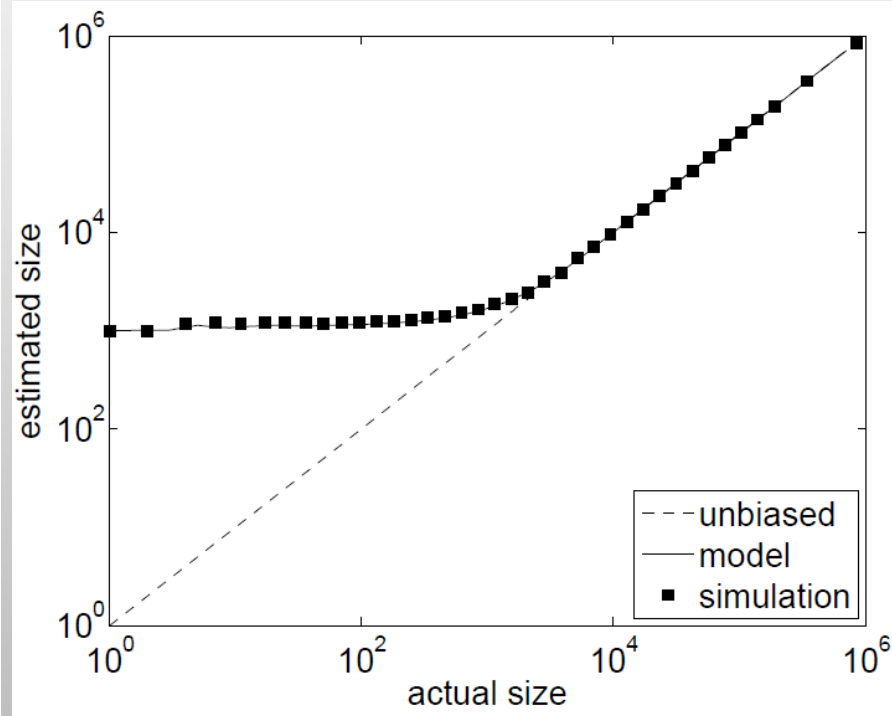
$$E[e(R_l)] = \frac{l}{1 - (1-p)^l}$$

- It tends to overestimate the original flow size by a factor of up to  $1/p$

# Simulations – Estimated Size



$p=0.01$



$p=0.001$

## Previous Method – Single-Flow Usage 2

- Quantifying the error of individual values  $e(R_l)$  in estimating flow size  $l$ 
  - Relative Root Mean Square Error (RRMSE)

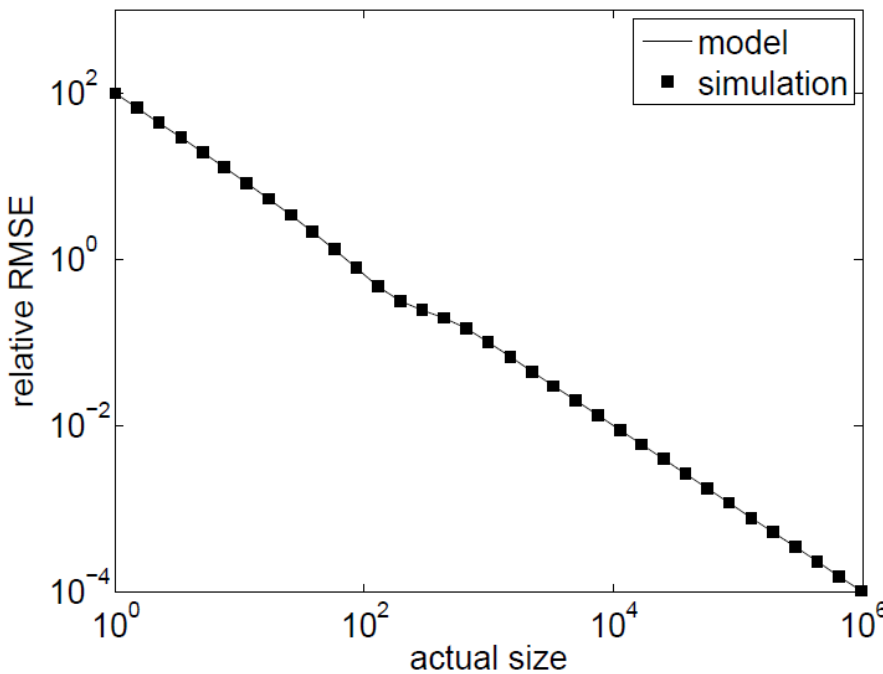
$$\delta_l = \sqrt{E[(Y_l - 1)^2]}$$

where  $Y_l = e(R_l)/l$  is relative error

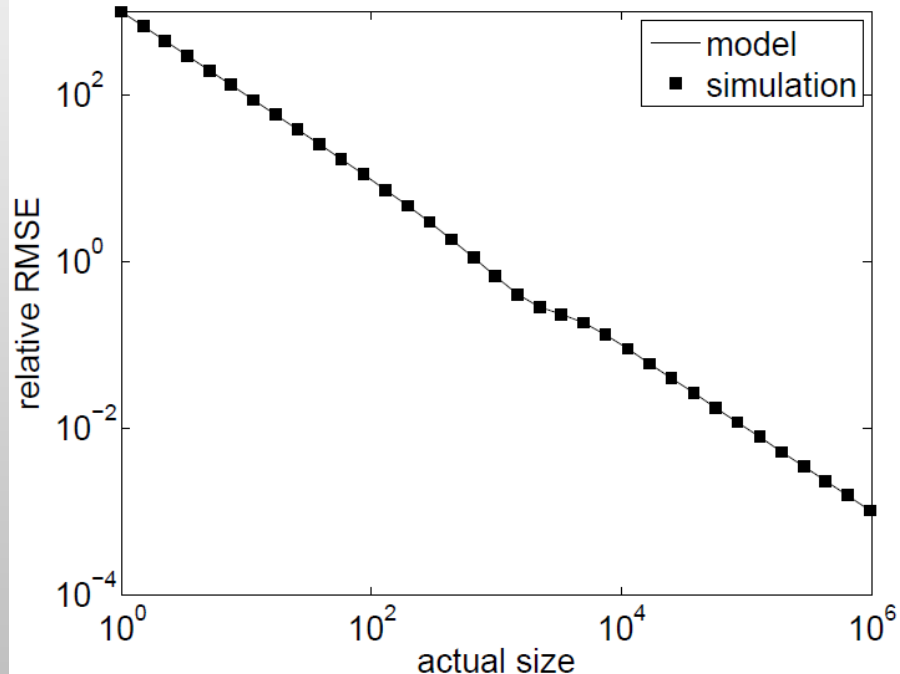
- *Theorem 2*: RRMSE of the existing RGS estimator:

$$\delta_l = \sqrt{\frac{1-p-l(l-1)p^2(1-p)^l - (1-p)^{l+1}}{l^2 p^2 (1-(1-p)^l)}}$$

# Simulations – Relative RMSE



$p=0.01$



$p=0.001$

## Previous Method – Flow-Size Distribution

- Consider PMF  $q_i$  of  $e(R_L)$  and compare it with  $f_i$   
$$q_i = P(e(R_L) = i)$$
- *Theorem 3*: PMF of flow sizes estimated from  $e(R_L)$

is:

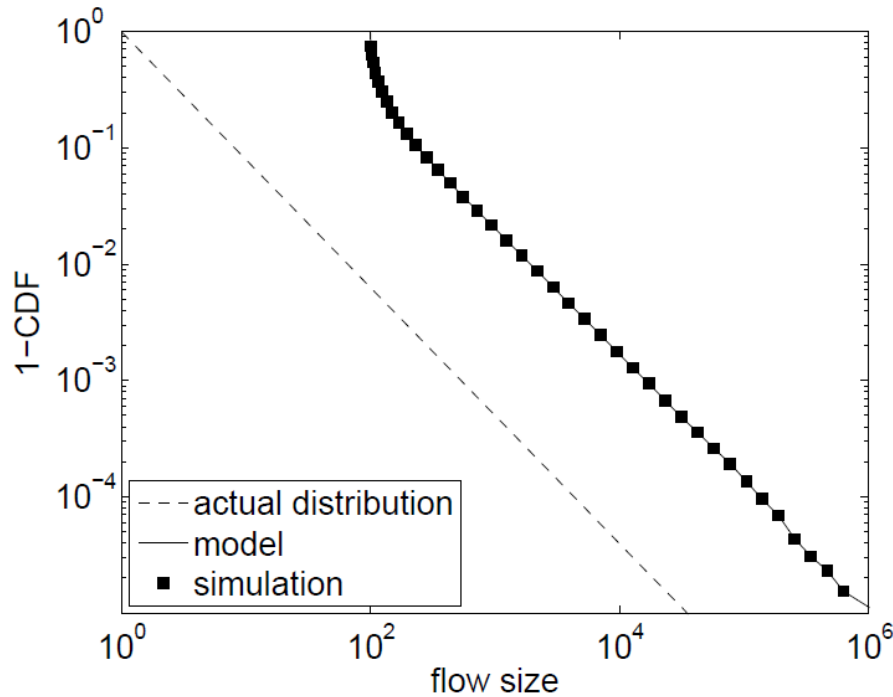
$$q_i = \frac{\sum_{j=y(i)}^{\infty} f_j (1-p)^{j-y(i)} p}{p_s}$$

– where  $p_s$  is the probability of a flow is selected and

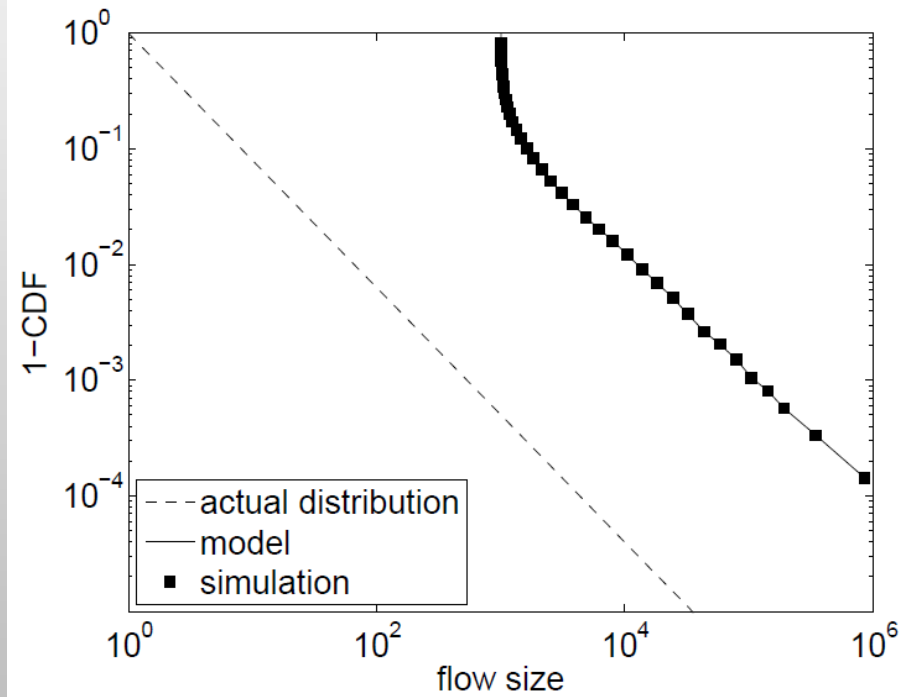
$$y(i) = \lceil i + 1 - 1/p \rceil$$

- The estimated distribution  $q_i$  is quite different from actual  $f_i$

# Simulations – Flow Size Distribution



$p=0.01$



$p=0.001$

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# URGE – Single-Flow Usage

- For single-flow size, we propose following estimator:

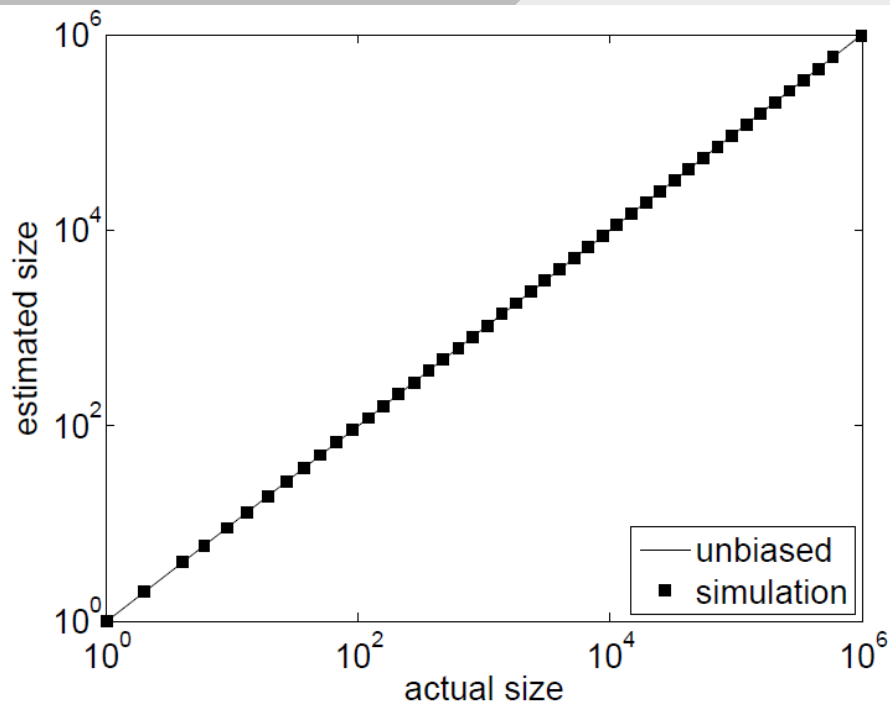
$$\hat{e}(R_L) = R_L - 1 + 1/p - \frac{(1-p)^{R_L}}{p}$$

→ same
→ different

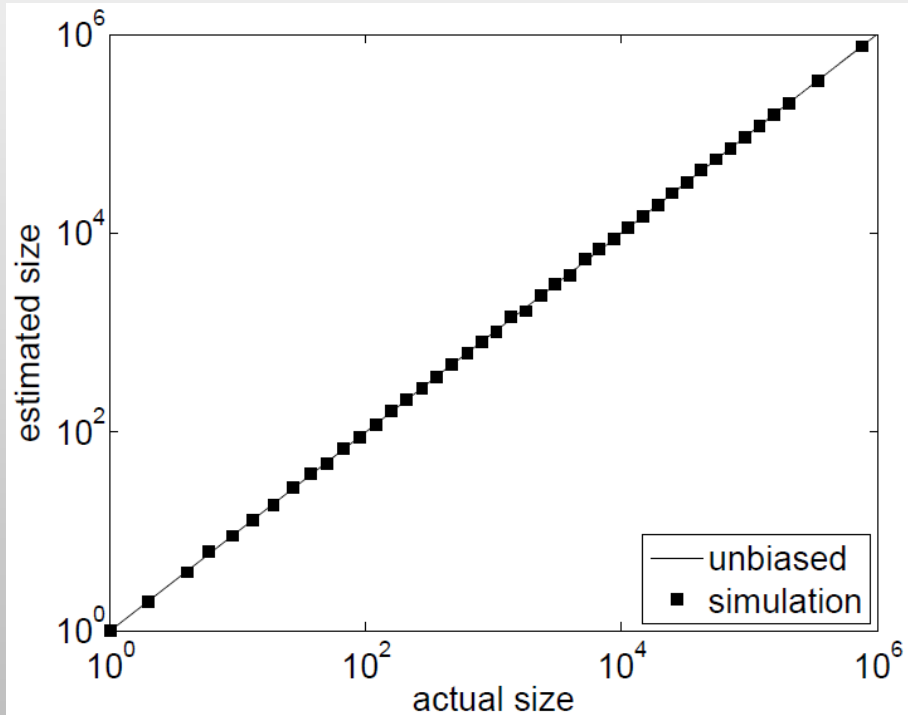
- Lemma 3:*  $\hat{e}(R_L)$  is unbiased for any flow size  $l$
- Theorem 4:* RRMSE of  $\hat{e}(R_L)$  as:

$$\hat{\delta}_l = \sqrt{\frac{1-p+lp(p-2)(1-p)^l - (1-p)^{2l+1}}{l^2 p^2 (1-(1-p)^l)}}$$

# URGE Simulations – Single-Flow Usage

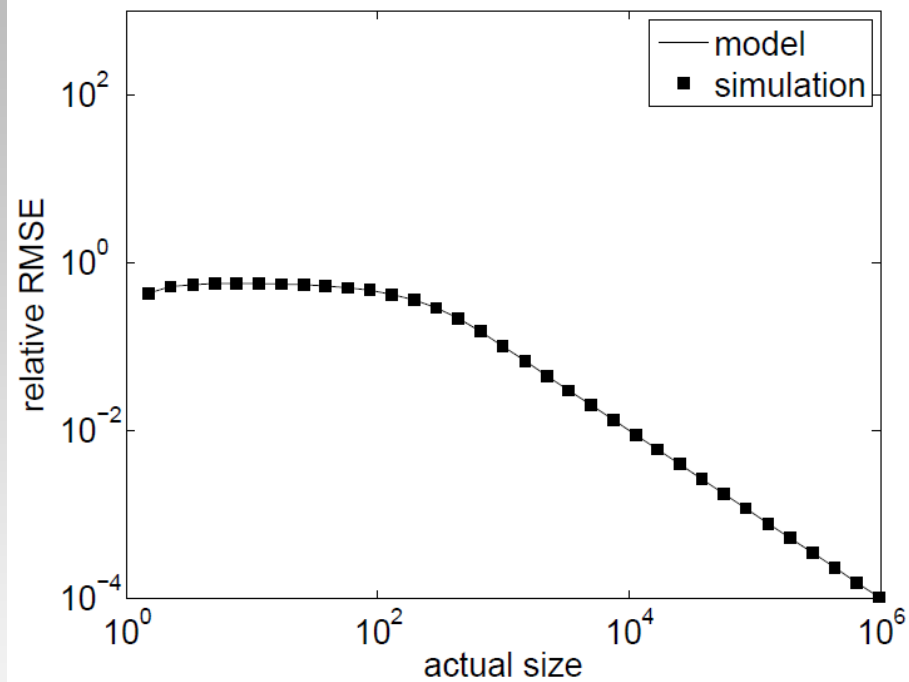


$p=0.01$

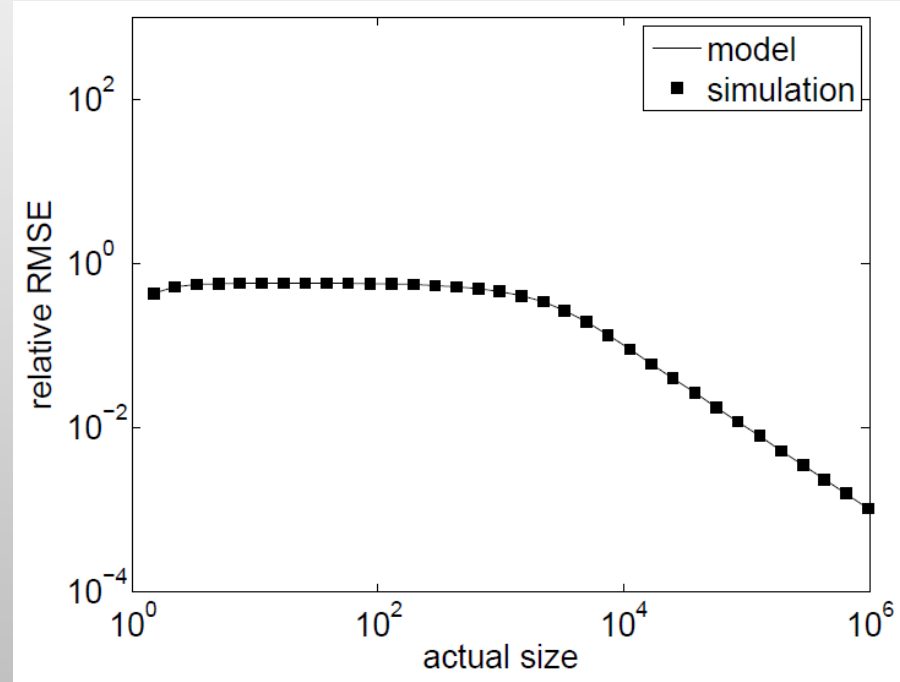


$p=0.001$

# URGE Simulations – Relative Error



$p=0.01$



$p=0.001$

# URGE – Flow Size Distribution

- *Lemma 5*: The flow size distribution  $f_i$  can be expressed using PMF of geometric residual  $h_i$  as:

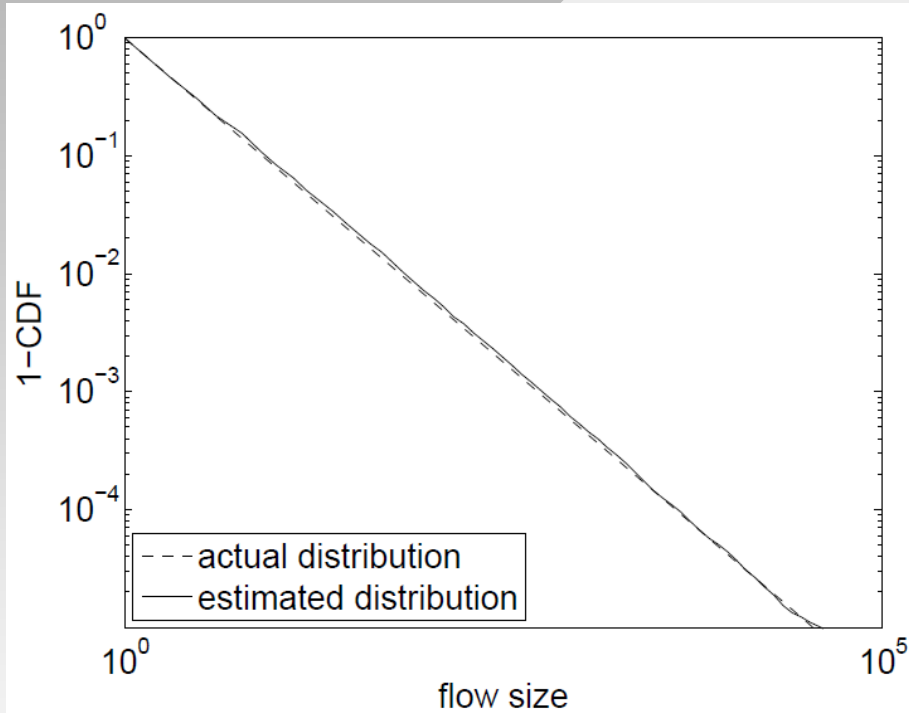
$$f_i = \frac{h_i - (1-p)h_{i+1}}{p + (1-p)h_1}$$

- For flow size distribution, we propose following estimator:

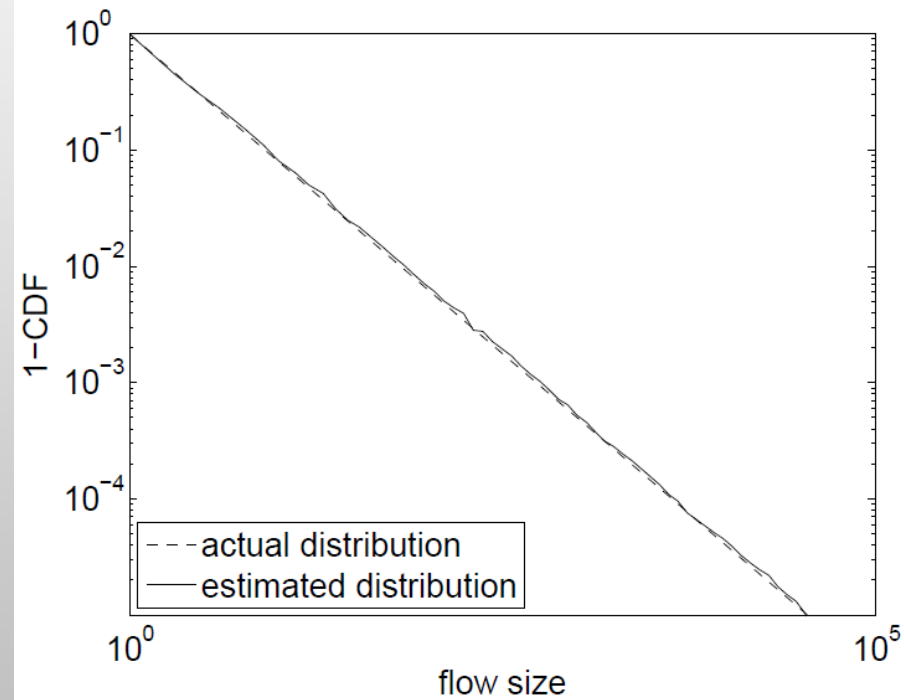
$$\tilde{q}_i = \frac{M_i - (1-p)M_{i+1}}{Mp + (1-p)M_1}$$

- *Corollary 2*: Estimator  $\{\tilde{q}_i\}$  is **asymptotically unbiased**, that is,  $\{\tilde{q}_i\}$  converges in probability to  $f_i$  as the number  $M$  of sampled flows  $\rightarrow \infty$

# URGE Simulations – Flow Size Dist.



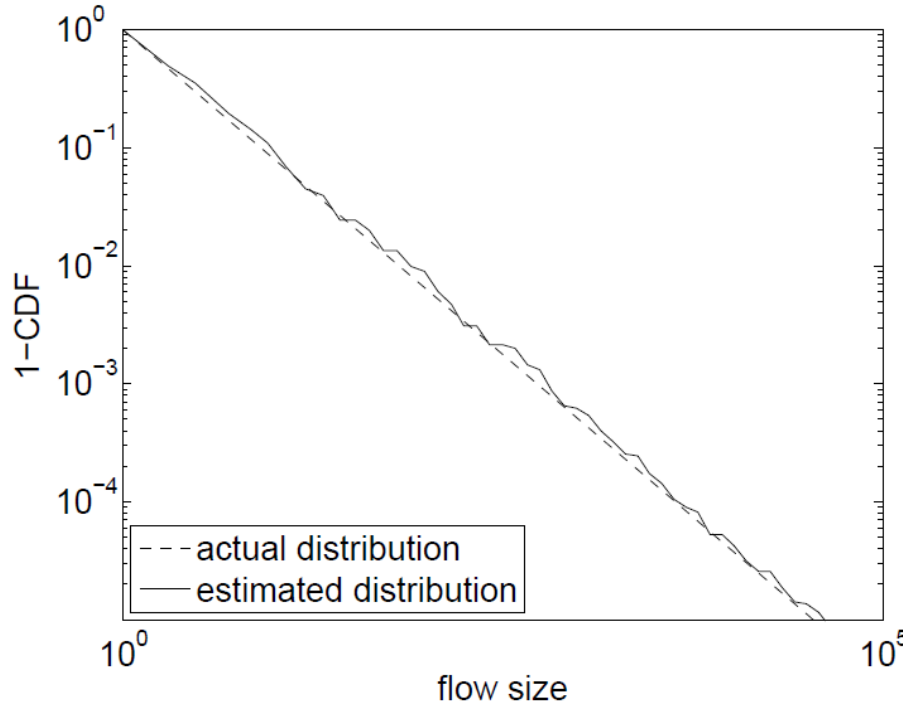
$p=0.01$



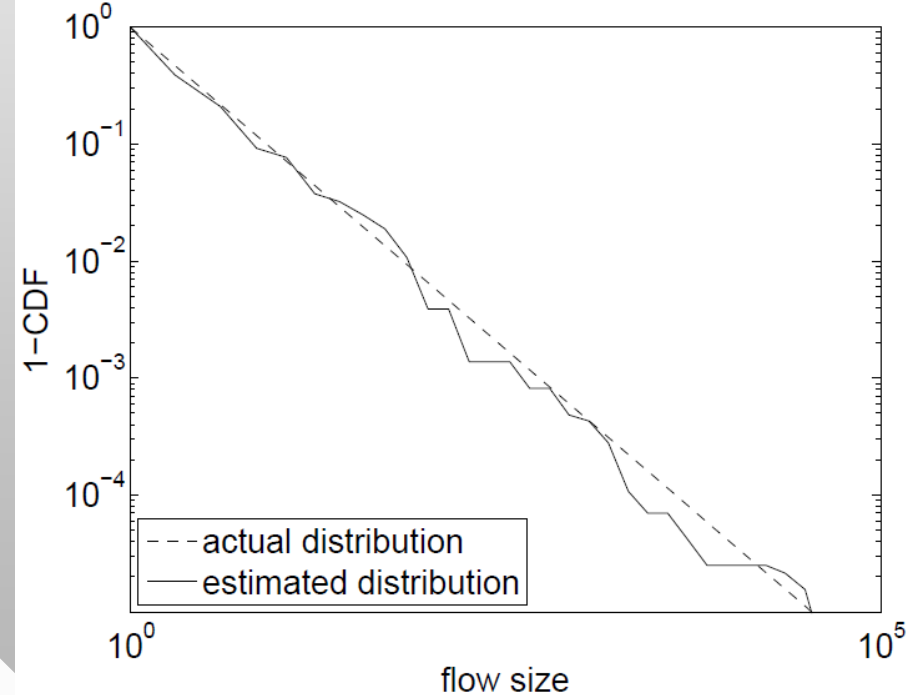
$p=0.001$

# URGE Convergence

- We next examine the effect of sample size  $M$  on the convergence of estimator



$$p=10^{-4}, M=3,090$$



$$p=10^{-5}, M=337$$

## URGE Convergence 2

- *Theorem 5:* For small constants  $\eta$  and  $\xi$  with probability  $1 - \xi$ , following holds for  $j \in [1, i+1]$

$$|\tilde{h}_j - h_j| \leq \eta h_j$$

if sample size  $M$  is no less than:

$$M \geq \frac{(1-h_i)}{h_i \eta^2} (\Phi^{-1}(1 - \xi/2))^2$$

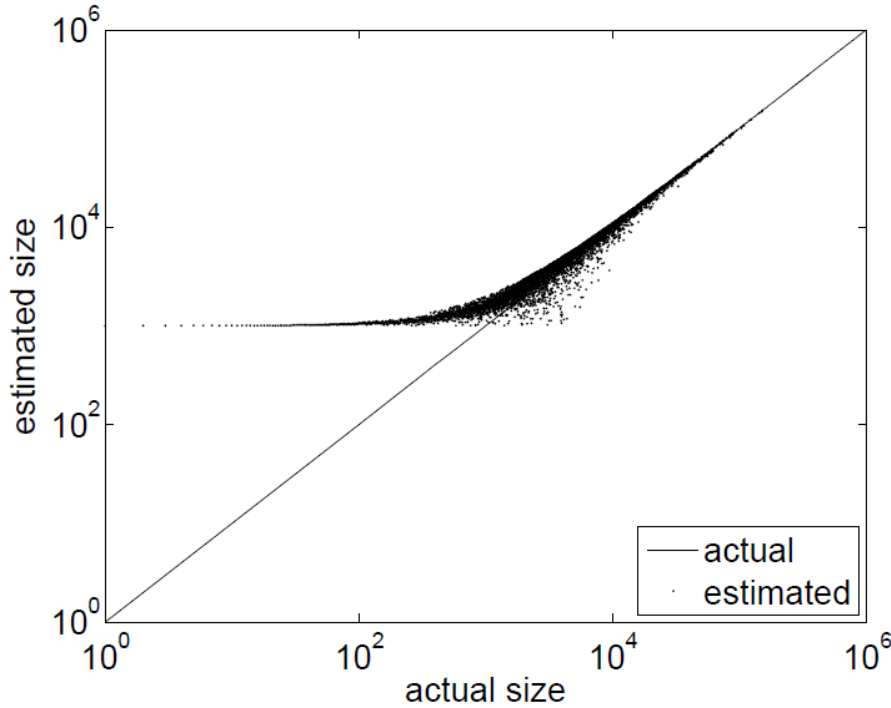
where  $\Phi(x)$  is the CDF of the standard Gaussian distribution  $N(0,1)$

# Performance Evaluation

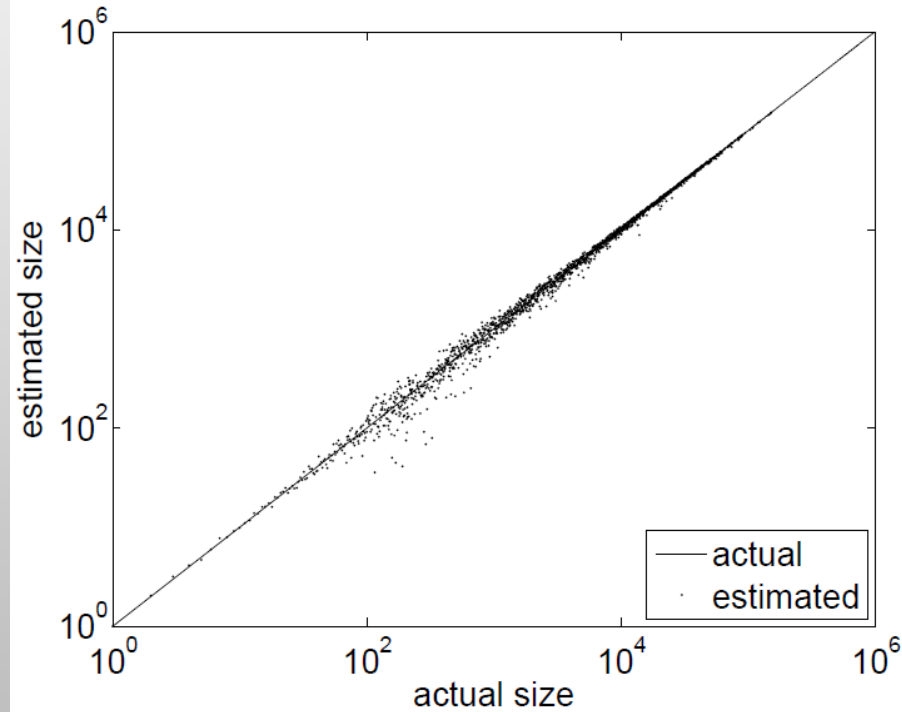
- We applied our estimation algorithm to the traces collected by NLANR and CAIDA
  - All of them confirm the accuracy of URGE
- As example, we show our experiment on dataset FRG, collected from a gigabit link between UCSD and Abilene



# Performance Evaluation – Usage

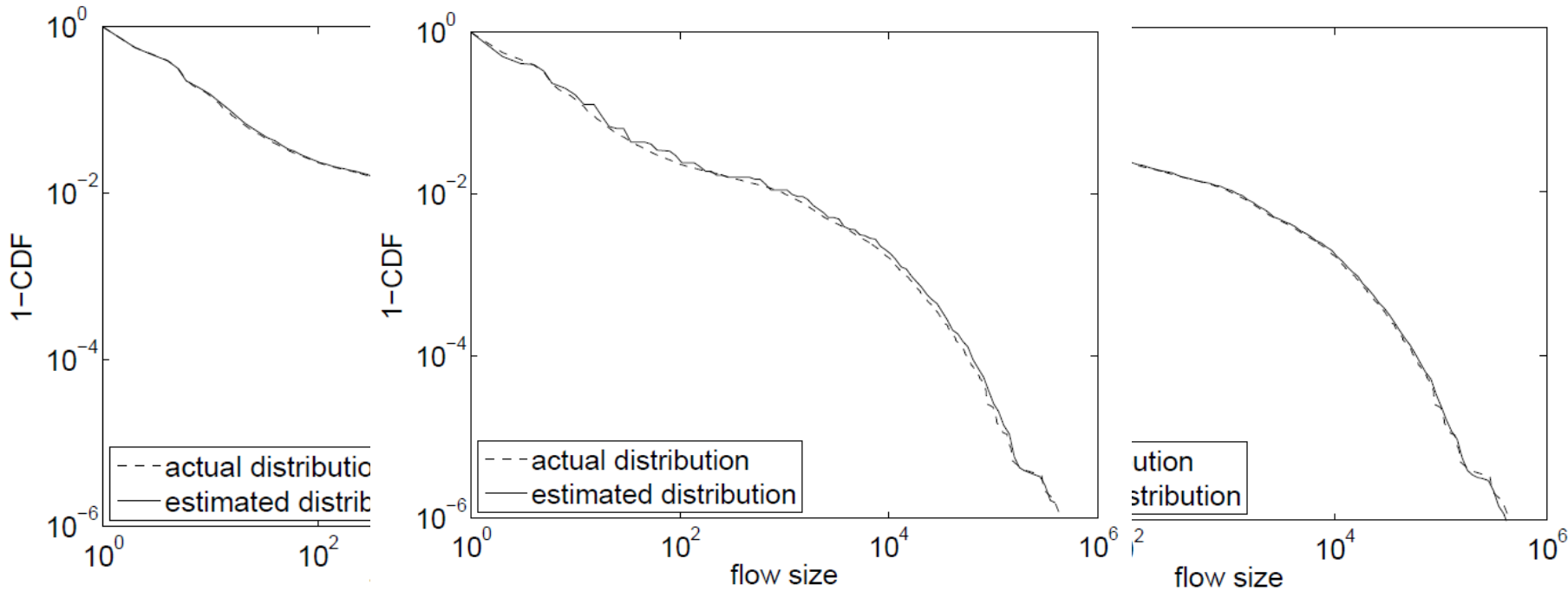


Previous method



URGE

# Performance Evaluation – Distribution



$p=0.01$

$p=0.0001$

$p=0.001$

## Conclusion

- We proposed a novel modeling framework for analyzing residual sampling
  - Proved that previous estimators based on RGS had certain bias
- We also developed a novel set of unbiased estimators
  - Verified them both in simulations and on Internet traces
- Results show that the proposed method provides an accurate and scalable solution to Internet traffic monitoring

# The End

- Thanks!