What Signals Do Packet-pair Dispersions Carry?

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Outline

• Introduction
• Characterization of Packet-pair Probing
  — “Sampling & construction” model
  — Statistical properties of probing signals
• Probing Response Curves
• Implication on Bandwidth Estimation
• Conclusion
Introduction 1

• Packet-pair probing has been a major mechanism to measure link capacity, cross-traffic, and available bandwidth.

• Due to its end-2-end nature of packet-pair measurement, no network support is needed.
• Unresolved questions in packet-pair measurements:
  — What information about the path is captured in the output packet-pair dispersions?
  — How are these signals encoded?
  — What are the statistical properties of these signals?

• Understanding these questions helps us extract path information from packet-pair dispersions.

• This paper answers these questions in the context of a single-hop path and bursty cross-traffic arrival.
Prior Work


\[
\delta' = \begin{cases} 
\frac{s}{C} & \delta \leq \frac{s}{C} \\
\delta & \delta > \frac{s}{C} 
\end{cases}
\]

- This becomes the basic idea for bottleneck capacity measurements.
Prior Work 2

- Single-hop path with constant-rate fluid cross-traffic. (Melander et al, Dovrolis et al)

\[
\delta' = \begin{cases} 
\frac{s}{C} + \frac{\lambda \delta}{C} & \frac{s}{\delta} \geq C - \lambda \\
\delta & \frac{s}{\delta} \leq C - \lambda 
\end{cases}
\]

\[
= \max \left( \delta, \frac{s + \lambda \delta}{C} \right).
\]

- In multi-hop paths, the same thing holds to a certain extent.
Prior Work 3

• Single-hop path with bursty cross-traffic
  — Bolot 1993, Hu et al 2003
  — When the packet-pair shares the same queuing period

\[ \delta' = \frac{s}{C} + \frac{y\delta}{C} \]

Output dispersion R.V.

— When \( \delta \) is sufficiently large (so that packet-pairs almost never share the same queuing period), the mean of the output dispersion is equal to \( \delta \).

The R.V. indicating cross-traffic intensity between the arrivals of the pair
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Sampling & Construction Model

A packet-pair inspects three random processes associated with the hop it arrives into and constructs the output dispersion signal $\delta_0$ based on the random sampling.
What are the random processes?

- The three processes which probing packet-pair inspects are all related to cross-traffic arrival.
- \( Y_\delta(t) \), \( \delta \)-interval cross-traffic intensity process, indicates the cross-traffic arrival rate in the time interval \([t,t+\delta]\).
- \( B_\delta(t) \), \( \delta \)-interval available bandwidth process, indicates the spare capacity in the time interval \([t,t+\delta]\).
What are the random processes? 2

- $D_\delta(t)$, $\delta$-interval workload difference process, is defined as

$$D_\delta(t) = W(t + \delta) - W(t)$$

- $W(t)$, workload process, indicates the remaining workload (in terms of the amount of service time) in the hop at time $t$. 
Construction Procedure

• A packet-pair constructs its output dispersion signal using the following formulas

\[
\delta' = \frac{Y_\delta(a_1)\delta}{C} + \frac{s}{C} + \max\left(0, \frac{B_\delta(a_1)\delta - s}{C}\right)
\]

\[
= \delta + D_\delta(a_1) + \max\left(0, \frac{s - B_\delta(a_1)\delta}{C}\right).
\]

The hop idle time between the departure of the pair \(\hat{I}_\delta(a_1)\)

Intrusion residual \(R_\delta(a_1)\)
Intrusion Residual $R_\delta(a_1)$

- $R_\delta(a_1)$ is the additional queuing delay imposed on the second probing packet by the first packet in the pair.

\[ \delta' = \delta + D_\delta(a_1) + R_\delta(a_1) \]
The advantage of our model

- The "sampling & construction" characterization of packet-pair probing holds unconditionally. It neither relies on any assumptions on cross-traffic arrival, nor imposes any restriction on input packet-pair dispersion $\delta$.
- Using this characterization, we answered fully the question as to what information is contained in output dispersions and how it is encoded.
Statistical Properties of Probing Signals

- To facilitate information extraction from $\delta'$, we examine the statistics of each encoded signal.
- Assumption: cross-traffic arrival has ergodic stationary increments.
  - $Y_\delta(t)$ has time-invariant distribution with ensemble mean $\lambda$ for any $\delta$ interval.
  - Ergodicity implies that the variance of $Y_\delta(t)$ decays to 0 when $\delta$ increases, for any $t$.

$$E[Y_\delta(t)] = \lambda.$$  $$\lim_{\delta \to \infty} \frac{E[(Y_\delta(t) - \lambda)^2]}{\delta} = 0.$$
As a consequence of our assumption (see details in the paper):
- Both $W(t)$ and $D_\delta(t)$ have time-invariant distributions.

\[
E[D_\delta(t)] = E[W(t + \delta)] - E[W(t)] = 0
\]

- $B_\delta(t)$ has a time-invariant distribution

\[
E[B_\delta(t)] = C - \lambda
\]

\[
\lim_{\delta \to \infty} Var[B_\delta(t)] = 0
\]
• Both $R_\delta(t)$ and $\tilde{I}_\delta(t)$ have time-invariant distributions, but their ensemble means depend on both $\delta$ and probing packet size $s$.

• Keeping $s$ fixed, we have

$$= \min \left( E[R_\delta(t)], E[\tilde{I}_\delta(t)] \right)$$
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**Probing response curve**

- Link capacity $C$, cross-traffic $\lambda$, and available bandwidth $C-\lambda$ are the pieces of information we are interested in extracting from packet-pair output dispersion random variable.

- This information is contained in $E[\delta']$ as a function of input dispersion $\delta$:
  - $E[\delta']$: the probing response of the path at input dispersion point $\delta$.

- The way to estimate $E[\delta']$ is to probe many times and generate an output dispersion random process $\{\delta'_n\}$:
  - The process has time-invariant distribution and its sample-path time-average is equal to $E[\delta']$. 
Closed-form expression for probing response curve

- Based on our “sampling & construction” model and stationary cross-traffic arrival assumption, we get

\[
E[\delta'] = \frac{\delta \lambda + s}{C} + \int_{s/\delta}^{C} \frac{x \delta - s}{C} dP_{\delta}(x)
\]

\[
= \delta + \int_{0}^{s/\delta} \frac{s - x \delta}{C} dP_{\delta}(x).
\]

\[
E[\tilde{I}_{\delta}(t)]
\]

\[
E[R_{\delta}(t)]
\]
Deviation from fluid response curve 1

- The two terms $E[\tilde{I}_\delta(t)]$ and $E[R_\delta(t)]$ cause the response curve to deviate from that in fluid cross-traffic, which complicates information extraction.

\[
E[\delta'_n] = \max \left( \delta, \frac{\delta \lambda + s}{C} \right) = \begin{cases} 
E[I_\delta(t)] & \frac{s}{\delta} \geq C - \lambda \\
E[R_\delta(t)] & \frac{s}{\delta} < C - \lambda 
\end{cases}
\]

Fluid response curve, where information can be easily extracted.

\[
= \min \left( E[R_\delta(t)], E[\tilde{I}_\delta(t)] \right)
\]
Deviation from fluid response curve 2

\[ E[\delta'] \]

Caused by \( E[\bar{I}_\delta(t)] \)

Caused by \( E[R_\delta(t)] \)

\[ \frac{s}{C} \quad \frac{s}{(C-\lambda)} \]
Deviation from fluid response curve 3

- Rate response curve is more convenient.

\[ r_i = \frac{s}{E[\delta']} \]

Caused by \( E[R_\delta(t)] \)

Caused by \( E[\tilde{I}_\delta(t)] \)
Deviation from fluid response curve 4

- A transformed version of rate response curve is even more convenient.

\[
\frac{r_I}{s / E[\delta']} \quad \text{Caused by} \quad E[\bar{I}_\delta(t)]
\]

\[
\text{Caused by} \quad E[R_\delta(t)]
\]

\[
\min\left(1, \frac{\lambda + r_I}{C}\right)
\]
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Computing response curves

• We proposed a method that computes $E[\delta']$ from cross-traffic arrival traces with high accuracy.
  — Given a trace, compute the sample-path $\delta'(t)$ in a time interval of the trace duration.
  — The sample-path $\delta'(t)$ is a piece-wise linear function, which allows accurate and easy computation of its time-average.
  — This time-average is a good approximation of $E[\delta']$ if the duration is sufficiently long.

• Alternatively, we can also measure the response using ns2 simulation.
Some results using Poisson CT 1
Some results using Poisson CT 2

The ratio between input and output rate $r'/r$.

- Real curve computed off-line
- Real curve measured in simulation
- TOPP-transformed fluid curve

Input rate $r$(mb/s)
Implication on two packet-pair measurement techniques

- TOPP uses the deviated portion of the response curve and produces inaccurate results.

<table>
<thead>
<tr>
<th></th>
<th>$C$</th>
<th>$\lambda$</th>
<th>$C-\lambda$</th>
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</thead>
<tbody>
<tr>
<td>Real Value</td>
<td>10</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>TOPP Ns-2</td>
<td>35.97</td>
<td>32.33</td>
<td>3.64</td>
</tr>
<tr>
<td>TOPP Off-line</td>
<td>35.81</td>
<td>32.38</td>
<td>3.43</td>
</tr>
</tbody>
</table>

- Spruce uses the curve at input rate $C$, where no deviation occurs. Hence, spruce is unbiased in single-hop path.

- However, Spruce is subject to significant under-estimation in multi-hop paths due to the two noise terms we discussed here. We report more details in the future work.
Recent progress (not in the paper)

- Using the "sampling & construction" model, we were able to show that the two noise terms converge in mean-square to 0 as packet-train length increases and that output dispersion $\delta'$ also converges in mean-square to the fluid response.

\[
\lim_{n \to \infty} E \left[ \left( \delta' - \max \left( \delta, \frac{s + \delta \lambda}{C} \right) \right)^2 \right] = 0
\]

- The trick is to treat the first and last packets in the train as a packet-pair, and treat probing packets in between as if they were from cross-traffic.
Conclusion

• We proposed a ``sampling & construction” model to characterize the signals contained in packet-pair dispersion.
• The presence of two positive-mean noise random signals impedes accurate information extraction from packet-pair output dispersions and response curves.
• The way to suppress the noise signals is to use large probing packet-size and long packet-trains instead of packet-pairs.
• Future work: extension to multi-hop paths.
Thank You!

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