On Node Isolation under Churn in Unstructured P2P Networks with Heavy-Tailed Lifetimes

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Agenda

• Motivation and background
  — Terminology, assumptions, and related work
• Generic node isolation model
• Max-age selection
• Age-proportional random-walk selection
• Wrap-up
**Terminology**

- **Resilience of unstructured P2P networks**
  - Ability of a network to remain connected under node failure, which is fundamental to system performance

- **User churn**
  - Each user stays in the system for $L$ random time units

- **Out-degree**
  - Joining users select $k$ neighbors

- **Neighbor replacement**
  - Detection of failed neighbors and replacement with existing peers occur within $S$ time units (can be fixed or random)
**Background**

Resilience

Global: disconnection of the graph

- Disconnection iff a node is isolated
  - Leonard 2005

Out-degree

- Joint in/out-degree
  - Yao 2006

Local: isolation of individual nodes before they depart

- Exponential lifetimes
- Heavy-tailed lifetimes
- No prior work

Real unstructured P2P networks
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Model Basics

- Neighbor residual lifetimes $R$
  - The time duration from the instance a peer is selected by user $v$ as a neighbor until the peer departs

- This metric depends on neighbor selection strategies
  - Some strategies may find users with large residual lifetimes with high probability while others may not
Model Basics 2

- Neighbor failure/replacement is an ON/OFF process

- Node out-degree evolution

\[ R: \text{residual lifetime} \]
\[ S: \text{search delay} \]
\[ W(t) \]
\[ T: \text{isolation time} \]
\[ k \]
\[ L: \text{user lifetime} \]

\[ P(T < L) \] isolation probability:
**Out-Degree Process**

- Determining the first-hitting time of $W(t)$ to zero (i.e., isolation time $T$) is difficult unless $W(t)$ is Markovian
  - Idea: replace the distribution of ON/OFF durations with a hyper-exponential approximation (see paper for details)
  - It is well-known (Feldmann 1998) that any completely monotone density function (e.g., Pareto, Weibull) can be approximated by a hyper-exponential PDF with arbitrary accuracy

- **Theorem 1**: For hyper-exponential neighbor residual lifetimes $R$ and hyper-exponential search delays $S$, the out-degree process $\{W(t)\}$ is a continuous-time Markov process
Node Isolation Probability

- Theorem 2: Given that \( \{W(t)\} \) is a Markov process, the PDF \( f_T(t) \) of the isolation time \( T \) can be obtained using the transition rate matrix of process \( \{W(t)\} \) shown in the paper.

- Then, it is straightforward to obtain:

\[
\phi = P(T < L) = \int_0^{\infty} P(L > t) f_T(t) dt
\]

node isolation probability

the CCDF of user lifetimes
Accuracy of Node Isolation Model

- Simulation results on isolation probability $\phi$ for $E[L] = 0.5$ hours and $k = 7$ under uniform selection

<table>
<thead>
<tr>
<th>$E[S]$ hours</th>
<th>Pareto $S$ with $\alpha = 3$</th>
<th>Weibull $S$ with $c = 0.7$</th>
<th>Exponential $S$</th>
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<td>Simulations</td>
<td>Model (15)</td>
<td>Simulations</td>
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<td>.001</td>
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<td>1.12 x 10^{-16}</td>
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<td>.8</td>
<td>7.78 x 10^{-3}</td>
<td>7.14 x 10^{-3}</td>
<td>1.12 x 10^{-2}</td>
</tr>
</tbody>
</table>

- Our model can be used to compute $\phi$ in networks with various types of lifetimes and different neighbor selection strategies
  - As long as the distribution of neighbor residual lifetimes can be approximated by a hyper-exponential distribution
Rules for Selecting Neighbors

- Higher resilience (i.e., smaller isolation probability) is achieved by selecting neighbors with larger residual lifetimes.
  - When it is impossible to obtain future knowledge of user remaining lifetimes $R$, user age $A$ may be used as a robust predictor of $R$.

- In systems with heavy-tailed lifetimes (e.g., Pareto, Weibull, and Cauchy), users with larger age demonstrate stochastically larger residual lifetimes.

- For light-tailed lifetimes (e.g., uniform distributions), it is the opposite.
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Basics of Max-Age Selection

• Suppose that each user $v$ publishes its joining time $t_v$ to its neighbors, so that they know $v$’s current age: $t - t_v$, where $t$ is the current time.

• Each user uniformly selects $m$ alive users at random from the system and chooses the one with the maximal current age as its neighbor.
  — Uniform selection can be implemented using special random walks on the graph (Zhong 2005)
  — When $m = 1$, the max-age approach reduces to the simple uniform approach.

• Denote by $U_m$ the residual lifetime of the user whose age is maximal among $m$ uniformly selected peers.
**Neighbor Residual Lifetimes**

- **Theorem 3**: For any heavy-tailed lifetime distribution, larger $m$ implies a stochastically larger neighbor residual lifetime $U_m$:
  
  $$P(U_m > x) \geq P(U_{m-1} > x), \quad m \geq 2$$

Simulation results on the tail distribution of $U_m$ for $\alpha = 3$

See the paper for the formula of the distribution of $U_m$

Open question: how does $m$ affect the obtained benefits?
Neighbor Residual Lifetimes 2

- **Theorem 4**: For Pareto lifetimes $L$ with CDF $F(x) = 1 - (1 + x/\beta)^{-\alpha}$, the mean residual lifetime $E[U_m]$ is proportional to $m^{1/(\alpha - 1)}$ for $\alpha > 2$ and non-trivial $m$
  - If $\alpha = 3$, $E[U_m] \sim \sqrt{m}$
  - For $\alpha \to 2$, the increase in $E[U_m]$ is more aggressive: $E[U_m] \sim m$
  - If $\alpha \leq 2$, the mean is infinite

- Max-age selection is much more effective in systems with more heavy-tailed lifetimes (e.g., smaller $\alpha$)
Node Isolation under Max-Age Selection

- By approximating the distribution of $U_m$ with a hyper-exponential distribution, we readily obtain isolation probability using our general node isolation model.

$$E[S] = 6 \text{ mins}$$

Isolation probability approaches 0 as $m \to \infty$.

$$y = 3E-05m^{-5.6941}$$

$R^2 = 0.9939$

$m = 6, \alpha = 3, \text{ and } k = 7$

$\alpha = 2, \text{ and } k = 7, E[S] = 6 \text{ mins}$
Discussion

• The max-age selection strategy requires sampling $m$ users per link
  — The overhead may not scale well for large $m$

• Much higher resilience can be achieved by more aggressively preferring users with large age

• We thus next propose a more efficient and effective neighbor selection strategy for heavy-tailed lifetimes
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Basics of Age-Proportional Selection

• We introduce a new age-biased neighbor selection method to ensure that the probability that user $v$ is selected by another peer is proportional to its current age $A_v$:

$$P(v \text{ is selected}) = \frac{A_v}{\sum_{u \in V} A_u}$$

the set including all existing users

• This approach is based on random walks on directed and weighted graphs
  — It provides a distributed solution that requires only one sample per link
Random Walks

• Assume that each user makes its current age and in-degree known to its in-degree neighbors
  — The weight of each link is determined by the current age and in-degree of the node that the link points to
  \[
  \text{weight} = \frac{\text{age of } v}{\text{in-degree of } v}
  \]

• Random walks are performed by alternating between walking along incoming and outgoing links
  — The probability that a link is chosen is proportional to its weight
  — The stationary distribution of the random-walk algorithm is:
  \[
  P(v \text{ is selected}) = \frac{A_v}{\sum_{u \in V} A_u} \quad \text{achieves the desired result}
  \]
Neighbor Residual Lifetimes

- **Theorem 5**: For random-walks where the above stationary distribution holds, the tail distribution of residual lifetimes $R$ of selected neighbors is:

$$P(R > x) = \frac{1}{E[L]E[A]} \int_0^\infty y(1 - F(x + y)) dy$$

  - the mean age
  - the user lifetime distribution

- For Pareto lifetimes $F(x) = 1 - (1 + x/\beta)^{-\alpha}$, $\alpha > 2$, the above yields:

$$P(R > x) = \left(1 + \frac{x}{\beta}\right)^{-(\alpha-2)}$$

  - The shape is reduced by 2

- The mean $E[R]$ is $\beta/(\alpha - 3)$ if $\alpha > 3$, and is infinite otherwise
Node Isolation Probability for $\alpha > 2$

- Simulation results on node isolation probability under age-proportional selection for Pareto $L$ and $k = 7$
  
  - Isolation probability is $10^4$ times smaller than that under uniform selection for $\alpha = 2.5$
  
  - Isolation probability converges to 0 as $\alpha \to 2$
  
  - In contrast, this metric under max-age selection does not tend to 0 unless $m \to \infty$ or $\alpha \to 1$ (both impossible to achieve in practice)

(a) $E[S] = 6$ minutes
Node Isolation Probability for $\alpha \leq 2$

- **Theorem 4**: For age-proportional random walks, Pareto lifetimes with $1 < \alpha \leq 2$, any number of neighbors $k \geq 1$, and any type of search delay (including $S = \infty$), as system age $\mathcal{T}$ and size $n$ converge to infinity, node isolation probability approaches:

$$\lim_{n \to \infty} \lim_{\mathcal{T} \to \infty} \phi = 0$$

- Gnutella has been shown to have $\alpha$ between 1.06 (Bustamante 2003) and 1.09 (Wang 2007)
  - These networks under age-proportional random walks approach an ideal system with zero node isolation probability as users join/depart the system.
Node Isolation Probability for $\alpha \leq 2$

- Simulation results of node isolation probability without replacing neighbors (i.e., $S = \infty$) for Pareto lifetimes

\[ \text{isolation probability} \]

\[ \begin{align*}
\text{system age (hours)} & \quad 1E+2 & \quad 1E+3 & \quad 1E+4 & \quad 1E+5 \\
\text{isolation probability} & \quad 1E-1 & \quad 1E-2 & \quad 1E-3 & \quad 1E-4 & \quad 1E-5 & \quad 1E-6 & \quad 1E-7 & \quad 1E-8 & \quad 1E-9 & \quad 1E-10 \\
\end{align*} \]

- monotonically decreases as system age increases

(a) $\alpha = 1.5$, $S = \infty$

(b) $\alpha = 1.2$, $S = \infty$
Wrap-up

• We developed a general node isolation model for any completely monotone density function of neighbor residual lifetimes
  — We applied this model to study node isolation behavior under uniform, max-age, or age-proportional random-walk selection to demonstrate its versatility

• We proposed a new neighbor selection strategy, age-proportional random walks
  — Under proposed neighbor selection, P2P networks with heavy-tailed lifetimes with $\alpha \leq 2$ become progressively more resilient over time and approach a system with zero node isolation probability, as more users join the system