Packet-Pair Bandwidth Estimation: Stochastic Analysis of a Single Congested Node

Seong-ryong Kang, Xiliang Liu,
Min Dai, and Dmitri Loguinov

Texas A&M University

City University of New York

Overview

- Motivation
- Definitions of Bandwidth
- Packet-pair bandwidth sampling
- Renewal cross-traffic
- Arbitrary cross-traffic
- Conclusion

Motivation

- Bandwidth estimation is an important area of Internet research
  - To understand the characteristics of network paths
  - Helps various Internet applications
- Majority of existing work is based on empirical studies
  - Assume no cross-traffic and/or
  - Based on fluid model
- Our work aims to provide stochastic insights on this field

Motivation 2

- Our purpose is not to offer another measurement tool
- Instead, we show that
  - Single-link case is completely tractable
  - Some of the existing methods cannot estimate bandwidth under heavy cross-traffic
- We also prove the existence of convergence for arbitrary cross-traffic

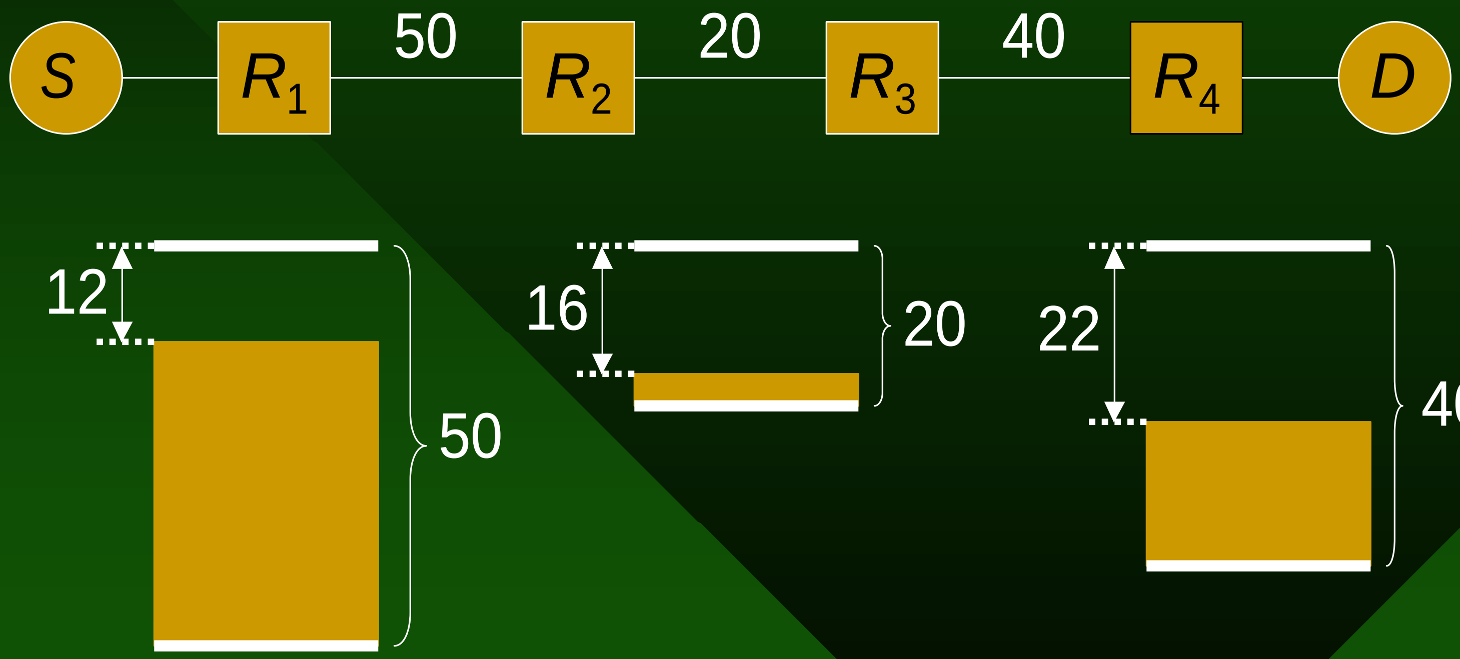
Bottleneck Bandwidth

- The capacity of the slowest link of an end-to-end path

- Bottleneck capacity: C = 20

Available Bandwidth

- The smallest average unused bandwidth along the end-to-end path



- Available bandwidth: $A = 12$

Available Bandwidth 2

- Multi-link case with arbitrary cross-traffic appears intractable at this stage
  - In this work, we restrict our analysis to a single link
- For an arbitrary cross-traffic arrival process $r(t)$, define the average rate of cross-traffic at a link

$$\bar{r} = \lim_{t \to \infty} \frac{1}{t} \int_0^t r(u)du$$

- Then, available bandwidth is defined as

$$A = C - \bar{r}$$

Packet-Pair Sampling

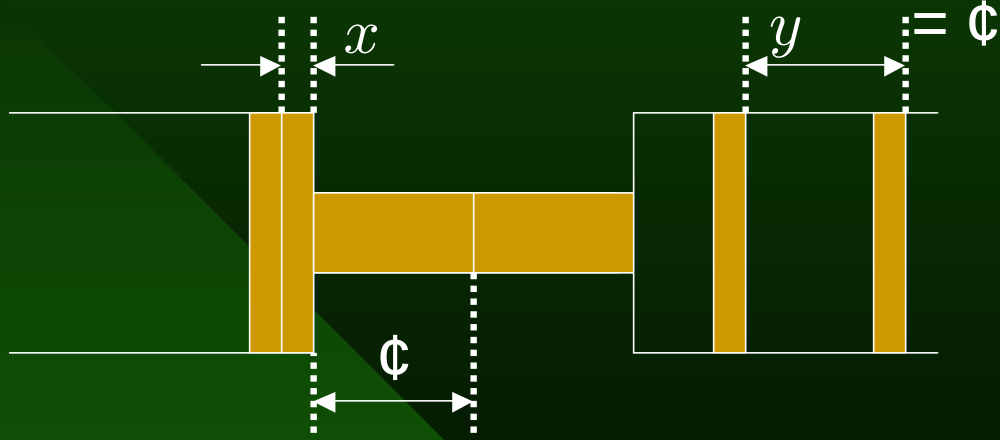
- Goal: measure both C and A over a single link with any cross-traffic arrival process

Packet-Pair Sampling 2

- Basic idea
  - Send back-to-back probe packets faster than C
  - Then, the probe packets are queued directly behind each other at the bottleneck link
  - The packet spacing between two probe packets are expanded due to transmission delay of the second packet at the bottleneck router
  - At the receiver, measure the inter-packet arrival spacing to estimate the capacity C

Packet-Pair Sampling 3

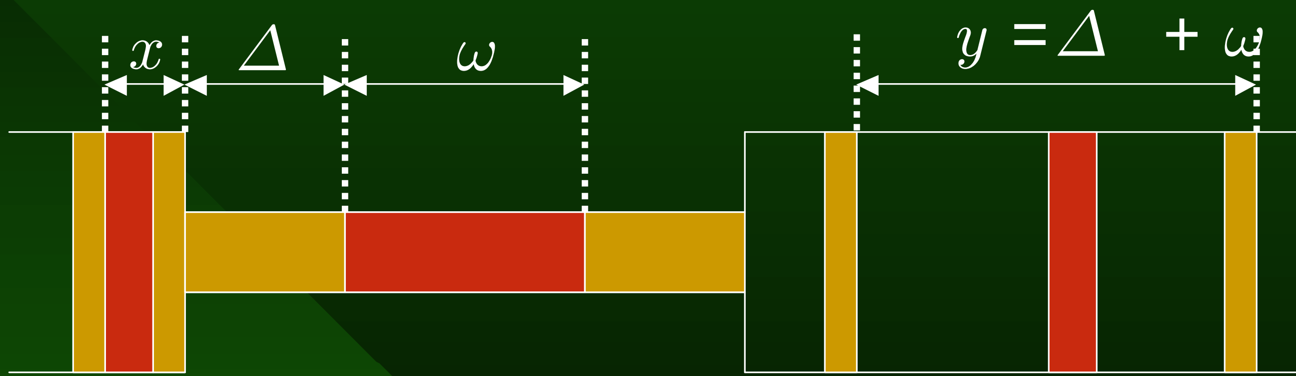
- Without cross-traffic, inter-packet arrival spacing is the same as the transmission delay ¢ of the second packet over the link



- Estimate C as q/y , (q is probe packet size)
- However, cross-traffic can lead to y ? Δ

Packet-Pair Sampling 4

- If cross-traffic packets arrive between two probe packets, inter-arrival spacing is expanded



- This leads to inaccurate estimation of C

$$\tilde{C} = q/y < q/\Delta = C$$

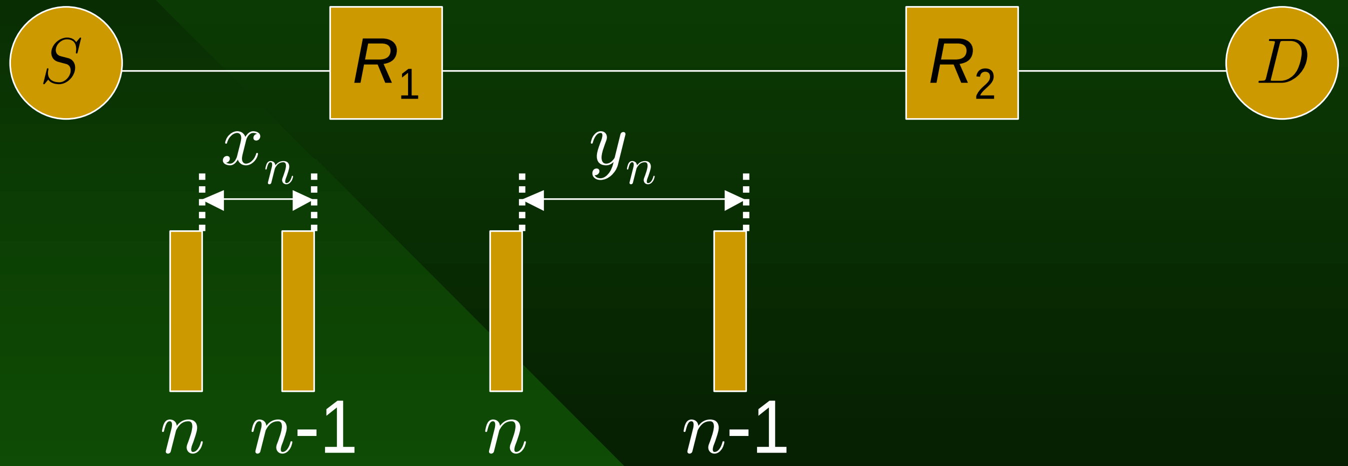
- Thus, filtering out the effects of cross-traffic noise is key for accurate estimation

Packet-Pair Sampling 5

- For bottleneck bandwidth estimation
  - Many existing studies apply various histogram-based methods
  - Assume no cross-traffic along the path
- For available bandwidth estimation
  - Cross-traffic is considered in the analysis
  - However, predominantly assumes fluid model for all flows
- In this work, a stochastic queuing model is used to analyze the random noise without fluid assumption

Stochastic Queuing Model

- Random process $x(n)$ is the initial spacing between n-th and $(n$-1)-th probe packets



- Inter-departure delay $y(n)$

$$y_n = \Delta + \omega_n$$

  – ω_n is random delay noise

Stochastic Queuing Model 2

- The distribution of $y(n)$ becomes fairly complicate without making prior assumption about cross-traffic
- Derive asymptotic results about process $y(n)$
- Note that $y(n)$ itself does not lead to any tractable results
  - Observation period of the process is very small
- Thus, define a time-average process W_n to be the average of $\{y_i\}$ up to time n:

$$W_n = \frac{1}{n} \sum_{i=1}^{n} y_i$$

Packet-Pair Analysis

- Assume ergodic renewal cross-traffic
  - Delays between cross-traffic packets are $i.i.d.$
- Claim 1: Time-average process W_n converges to:

$$\lim_{n\to\infty} W_n = E[y_n] = \Delta + \frac{\lambda E[x_n] E[S_j]}{C} = \Delta + E[\omega_n]$$

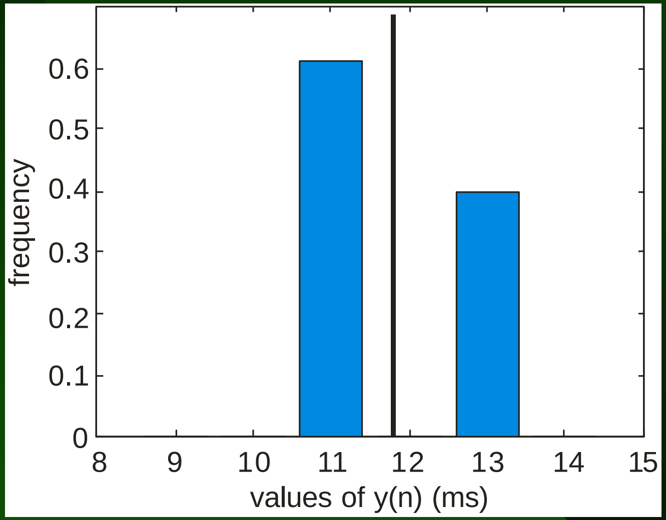
arrival rate of cross-traffic (λ)

size of cross-traffic packets (S_j)

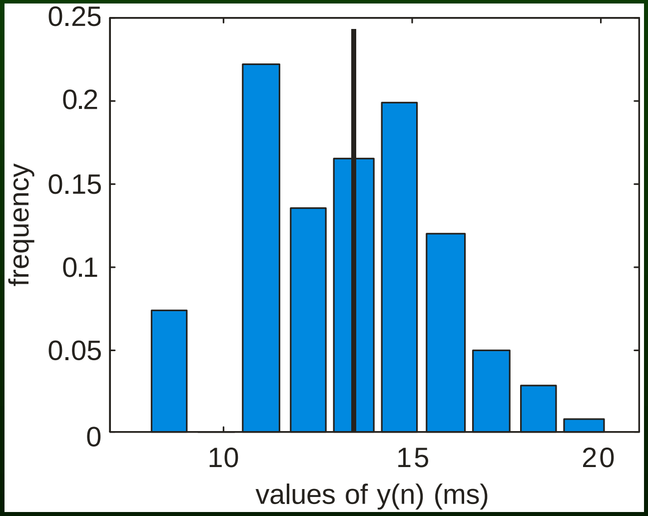
random delay noise (ω_n)

Packet-Pair Analysis 2

- Histogram of measured inter-arrival times y_n
  - □ C = 1.5 mb/s (¢ = 8 ms), $\bar{r} = 1$ mb/s



CBR cross-traffic

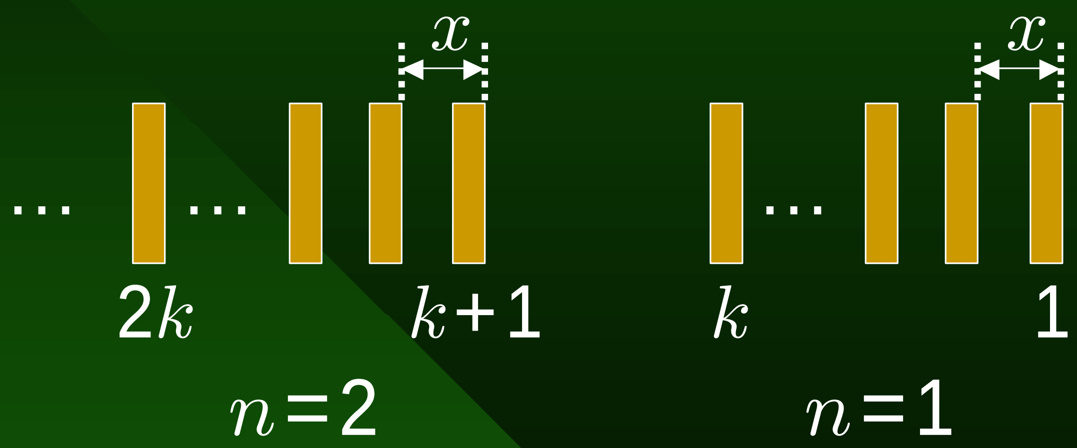


TCP cross-traffic

- None of CBR samples are located at ¢
- Mean of sampled signal W_n is shifted from ¢

Packet-Train Analysis

- What is a packet-train?
  - Bursts of probe packets sent back-to-back



- □ n is burst number
- □ k is the size of packet-train, which is the number of packets sent at a single burst n

Packet-Train Analysis 2

- Some studies suggested that packet-train measurements converge to the available bandwidth
  - By Carter *et al*. (1996) and Ahlgren *et al*. (1999)
  - No analytical evidence to this effect has been presented so far
  - Is this really true?
- Other studies used packet-train estimates to increase the measurement accuracy
  - Dovrolis *et al*. (INFOCOM 2001)
  - Not clear how these samples benefit estimation process

Packet-Train Analysis 3

- Next, we examine packet-train methods
  - Provide statistical insights on this technique
- Define packet-train samples as the average of inter-packet arrival delays within each burst n

$$\{Z_n^k\} = \frac{1}{k-1} \sum_{j=2}^{k} y_{k(n-1)+j}, \quad k \geq 2$$

Packet-Train Analysis 4

- Next assume renewal cross-traffic
- Claim 2: For sufficiently large k, constant $x_n = x$, and regenerative arrival process of cross-traffic, packet-train samples converge to Gaussian distribution for large n:

$$\{Z_n^k\} \xrightarrow{D} N\left(\Delta + \frac{\lambda x E[S_j]}{C}, \frac{\lambda x Var[S_j - \lambda E[S_j] X_i]}{(k-1)C^2}\right),$$

mean

$= E[y_n]$

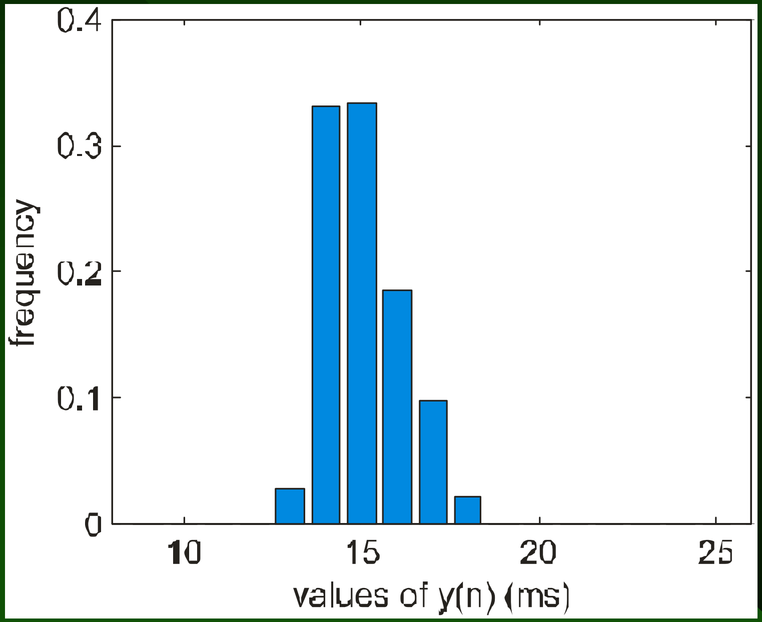
variance

→ tends to zero for large k as long as Var[X_i] is finite

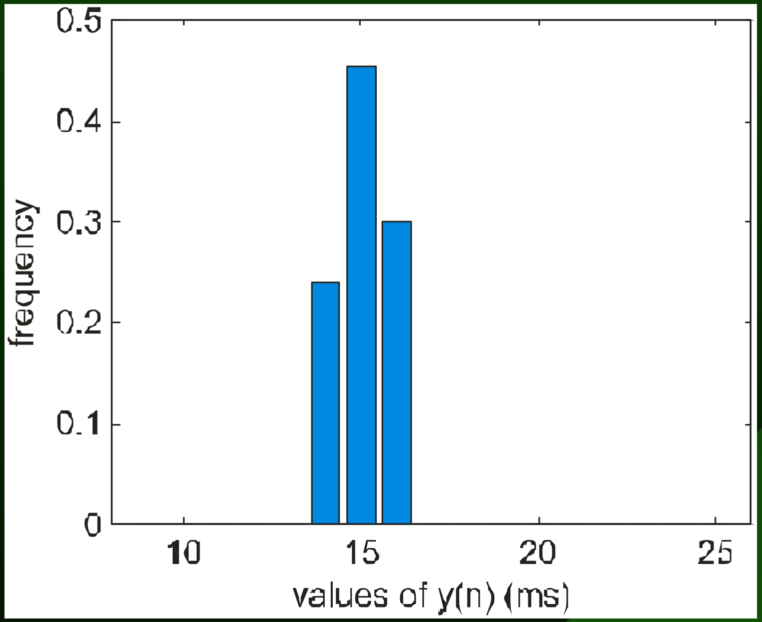
Inter-arrival time of cross-traffic

Packet-Train Analysis 5

- Histograms of measured inter-arrival times based on packet-trains with burst lengths k



$k = 5$



$k = 10$

Packet-Train Analysis 6

- Our results in Claim 2 offer statistical explanation for prior findings (e.g., Dovrolis *et al*. INFOCOM 2001) :
  - The histogram of packet-train samples becomes unimodal with increased k
  - The distribution of packet-train samples exhibits lower variance as packet-train size k increase
  - Packet-train histograms for large k tend to a single mode whose location is “independent of burst size k
- However, there is no evidence that packet-train samples measure the available bandwidth
- Deeper analysis is in our IMC 2004 paper

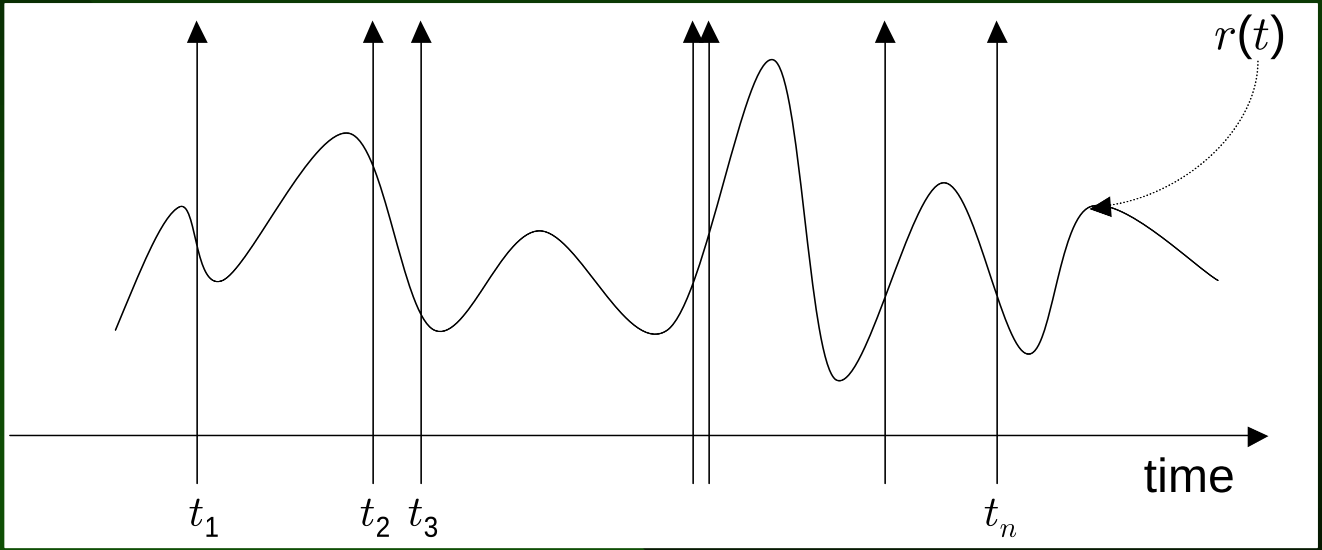
Arbitrary Cross-Traffic

- Observe that neither the $i.i.d.$ assumption nor stationarity holds for regular Internet traffic
- Thus, we build another model using PASTA principles
  - Restricts sampling process, but works with arbitrary cross-traffic
- Only assumption we impose on cross-traffic is the existence of its finite time-average

$$\bar{r} = \lim_{t \to \infty} \frac{1}{t} \int_0^t r(u)du < \infty$$

Arbitrary Cross-Traffic 2

- PASTA is based on Poisson sampling
  - Sample with $i.i.d.$ exponential random delays

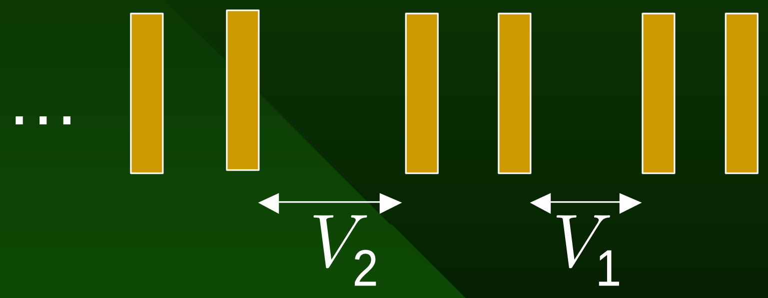


- The average of $r(t_i)$ converges to $\bar{r}$

$$\lim_{n\to\infty} \frac{r(t_1) + r(t_2) + \ldots + r(t_n)}{n} = \lim_{t\to\infty} \frac{1}{t} \int_0^t r(u)du = \bar{r}$$

Arbitrary Cross-Traffic 3

- In actual probing, Poisson sampling is achieved by sending packet-pairs with exponential intervals



  - Metric V_i is an exponential random variable

Arbitrary Cross-Traffic 4

- It can be shown that time-average process W_n converges to:

$$\lim_{n \to \infty} W_n = \Delta + \frac{x\bar{r}}{C}$$

- Notice that the above equation is a linear function of x
  - ¢ is the intercept and $\bar{r}/C$ is the slope

Arbitrary Cross-Traffic 4

- We next separate $\bar{r}/C$ from ¢
  - Use two sets of measurements $\{y_n^a\}$ and $\{y_n^b\}$ with two different spacings x_a and x_b
- Claim 4: For a single congested bottleneck with finite time-average rate, the estimate of ¢ at time n converges to ¢:

$$\lim_{n\to\infty} \tilde{\Delta}_n = \lim_{n\to\infty} \left(W_n^a - x_a \frac{W_n^a - W_n^b}{x_a - x_b} \right) = \Delta$$

time-average of $\{y_n^a\}$

time-average of $\{y_n^b\}$

Arbitrary Cross-Traffic 5

- From claim 4, estimated capacity $\tilde{C}_n$ converges to C:

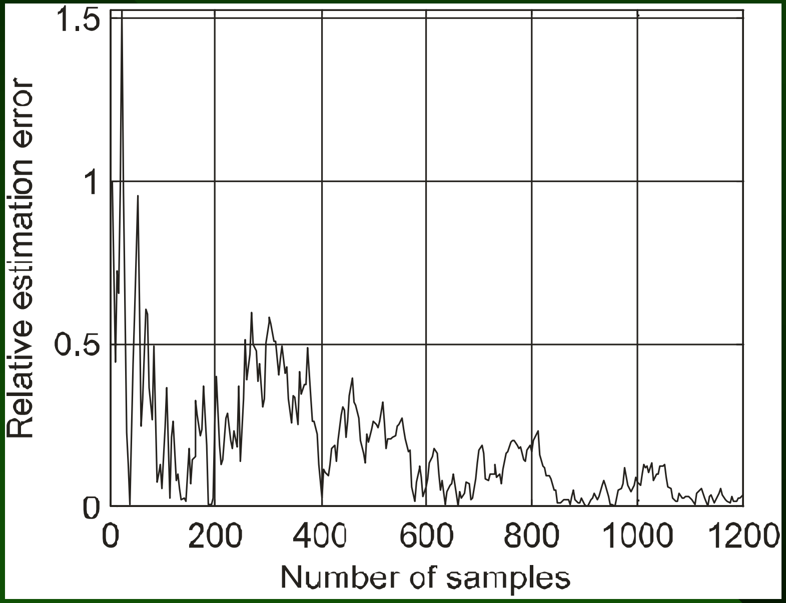
$$\lim_{n\to\infty} \tilde{C}_n = q/\tilde{\Delta}_n = \lim_{n\to\infty} \frac{q(x_a - x_b)}{x_a W_n^b - x_b W_n^a} = C$$

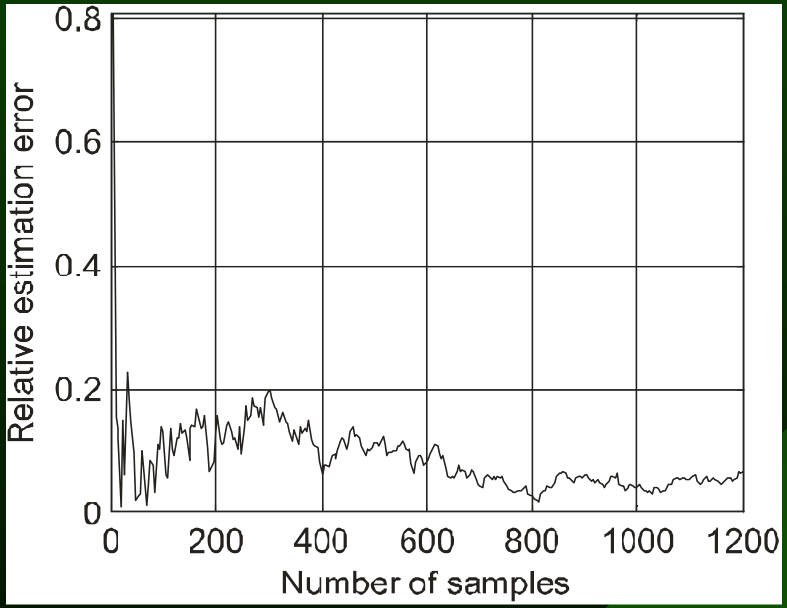
- Also, the following estimates of available bandwidth converge to A:

$$\lim_{n\to\infty} q\left(\frac{x_a - x_b - W_n^a + W_n^b}{x_a W_n^b - x_b W_n^a}\right) = C - \bar{r} = A.$$

Arbitrary Cross-Traffic 6

- Evolution of estimation errors with C=1.5 mb/s and 85% link utilization





$$\frac{|\tilde{C} - C|}{C}$$

$$\frac{|\tilde{A} - A|}{A}$$

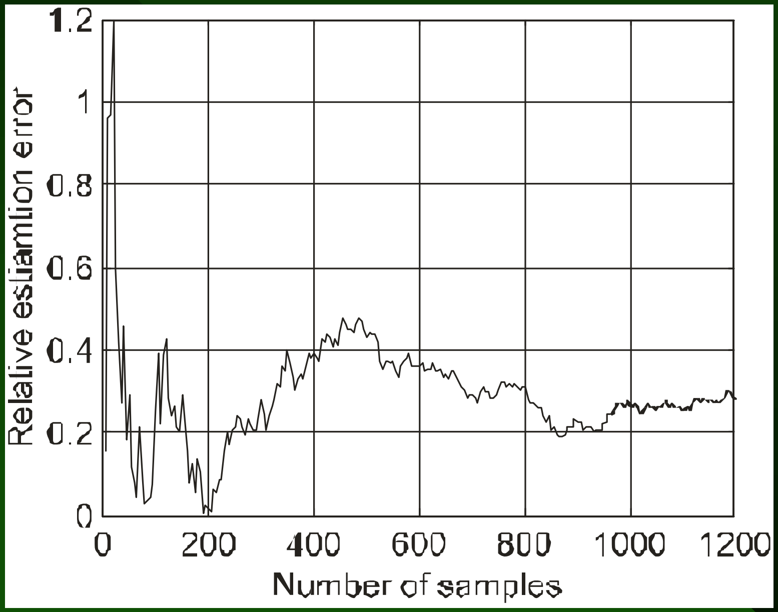
Arbitrary Cross-Traffic 7

- Compare available bandwidth estimation errors

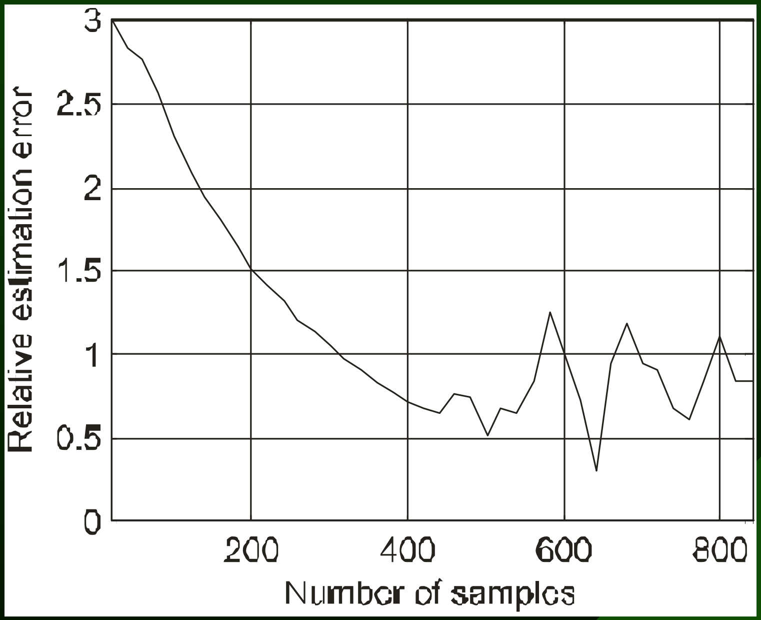
Bottleneck capacity	Relative error			
C (mb/s)	Ours	Pathload	Spruce	IGI
1.5	8.6%	46.5%	27.9%	84.5%
5	8.3%	40.1%	23.4%	90.0%
10	10.1%	40.9%	26.9%	89.0%
15	7.7%	38.5%	24.5%	83.1%

Arbitrary Cross-Traffic 8

- Relative estimation errors produced by Spruce and IGI with C=1.5 mb/s and 85% link utilization



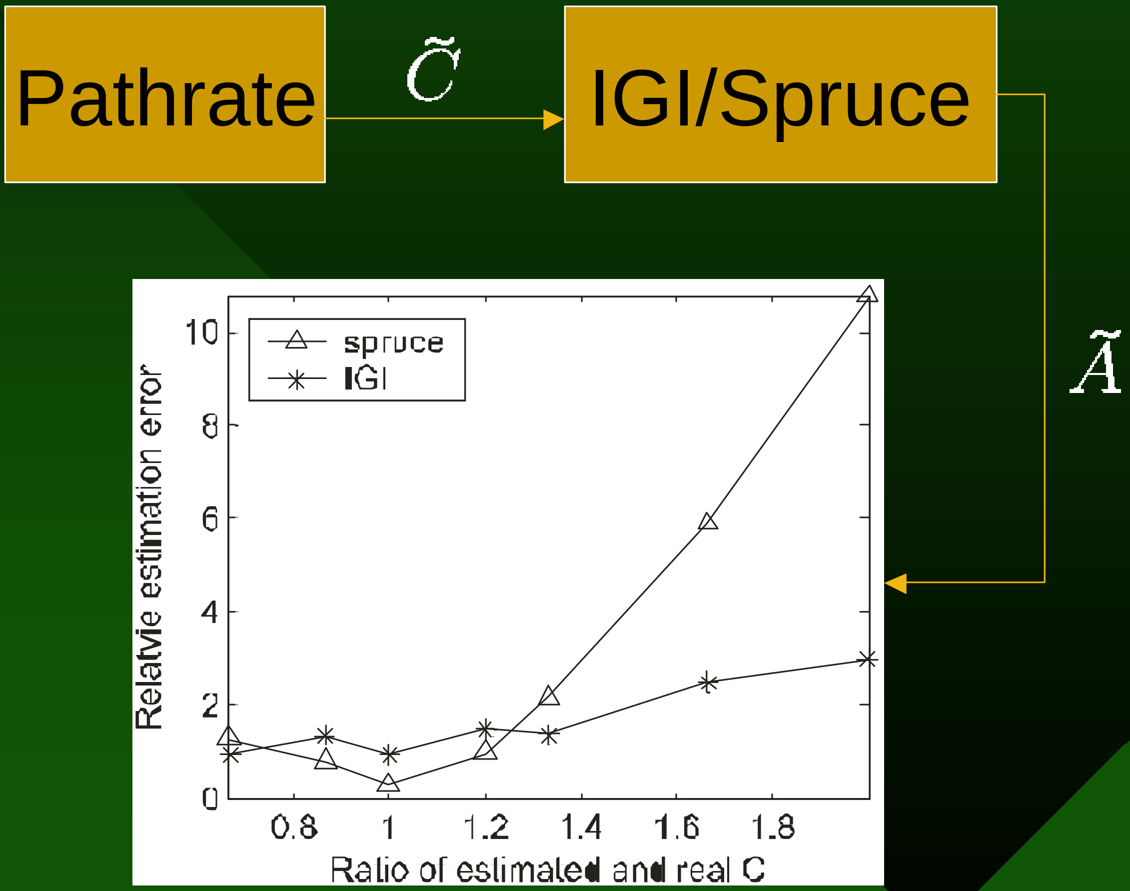
Spruce



IGI

More on Spruce and IGI

- Notice that Spruce/IGI require prior knowledge about bottleneck capacity C



Conclusion

- Single-node case is tractable with stationary renewal cross-traffic and arbitrary sampling
  - It is also tractable under arbitrary cross-traffic and Poisson sampling
  - Both C and A can be estimated simultaneously
- Multi-link appears difficult
- Low-rate sampling and deeper stochastic analysis of existing methods are in our IMC 2004 paper